Electricity and Magnetism I

Department of Physics

Abstract

This is an undergraduate course for students in

Contents

Chapter 1

Concept of Charge

SPH101 Electricity and Magnetism I

Lecture No. 1.
Outline

concept of charge

Coulomb's law

properties of electric

charge the electric eld

Concept of charge. The Greek philosophers, as early as 600 BC, knew that if you rubbed amber it could pick up pieces of straw. They also knew that some naturally occurring stones which we now know as mineral mag-netite would attract iron. These are some of the early and modest origins of the concept of charge and charged bodies. The science of electricity and magnetism, and later electromagnetism, developed from these origins. Some of the early pioneers of this subject in physics include Hans Christian Oer-sters, Michael Faraday, James Clark Maxwell, Charles Augustin Coulomb and later developments by Oliver Heaviside, H. A. Lorentz, Heinrich Hertz, Albert Einstein, Steven Weinberg, Abdus Salam and Sheldon Glashow.

If you walk across a carpet in dry weather, you can draw a spark by touching a metal door knob. On a large scale, lightning is familiar to every one. Such phenomena suggest vast ammounts of electric charge that is stored in familiar objects arround us. Generally most bodies arround us have balanced charges hence the electrical neutrality. A body is said to be charged when there is charge imbalance within the body. Charged bodies exert forces on each other.

Illustrations: A glass rod can be charged by rubbing it with silk. Rub-bing causes transfer of small amounts of charge between the bodies thus upsetting the electrical neutrality in each of the bodies. If two such charged rods are suspended by a thread and brought close, they would repel each other. However, if you rub a plastic rod with fur, it attracts the charged end of the glass rod as illustrated on Figure 1.1. This illustrates the following concepts,

1. there are two kinds of charges - positive and negative, and
2. like charges repel each other while unlike charges attract each other.

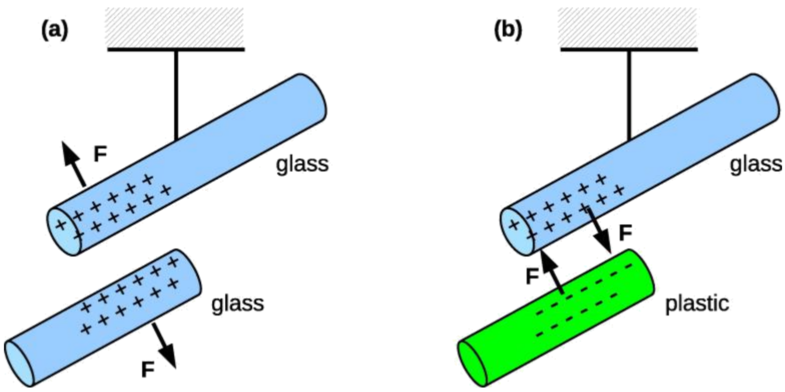

Figure 1.1: An illustration of the concept of charge (a) two positively charged glass rods repel each other and (b) a positively charged glass rod and a negatively charged platisc rod attract each other.

Electrostatics is the study of charges that are either at rest with respect to eacht other or moving very slowly. Conductors allow the ow of charge easily whereas non-conductors do not allow ow of charges. Example of conduc-tors are metals, tap water, human body etc. Non-conductors include glass, chemically pure water, plastics, etc.

Assignment 1: What are semi-conductors?

Coulomb's law. Experiments by Charles Ausgustine Coulomb (1736 - 1806) and his contemporaries showed that the magnitude of the electrical force F exerted by a body on another depends directly on the product of the two charges q_1; q_2 and inversely proportional to the square of their separation distance r, i.e.

$$F / \frac{q_1 q_2}{r^2}$$

or,

$$F = k \quad \frac{q_1 q_2}{r^2} \qquad (1.1)$$

The Equation (1.1) is called Coulomb's law and it holds for charged bodies whose sides are much smaller that the distance between them, i.e. point charges.

In the SI system, the constant k is expressed in the following form,

$$k = \frac{1}{4 \quad _0} \qquad (1.2)$$

where ’ 8:85 10 $^{12}C^2$=Nm^2 is the permittivity constant. Therefore, the constant k takes the value k ’ 8:99 10^{9}Nm^2=C^2.

Assignment 2: Compare Coulomb’s law and Newton’s law of gravitation,

$$F = G\frac{m_1 m_2}{r^2}$$

which was already more that 100 years old at the time of the experiments by Coulomb.

Coulomb’s law in vector form. Consider two point charges q_1 and q_2 separated by a distance r_{12} such that, F_{12} is the force exerted on the charge q_1 by charge q_2, and, r_{12} is the distance between the two point charges as illustrated on Figure (1.2).

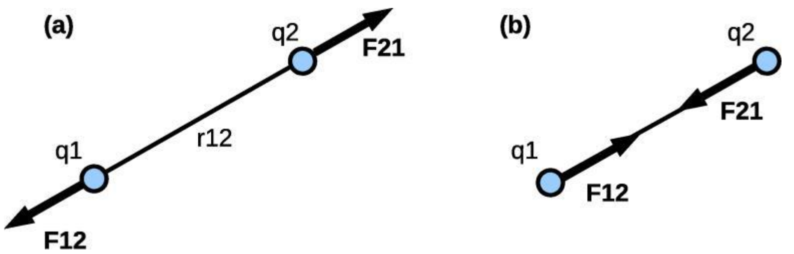

Figure 1.2: The force experienced by (a) two point charges of like charges and (b) two point charges of unlike charges.

The position vector of the point charge q_1 relative to the q_2 is denoted as r_{12}, with a unit vector $\hat{r}_{12}$. If the two point charges are have the same sign then the force F_{12} is repulsive and must be parallel to r_{12}. If the charges have opposite charges, then the force F_{12} is attractive and anti-parallel to r_{12}. In either case, the force is represented as,

$$F_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} \qquad (1.3)$$

where r_{12} is the magnitude of r_{12}.

Similarly, the fore exerted on the point charge q_2 by q_1 is,

$$F_{21} = k \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} : \qquad (1.4)$$

The vector form of Coulomb's law carries directional information of the forces and whether they are attractive or repulsive. It is very useful when describing the forces acting on an assembly of more than two charges. In such cases, Equation (1.3) would hold for each pair of charges. The total force on any one charge would then be found by taking the vector sum of the forces due to each of the other charges. For example, the net force acting on a charge q_1 in an assembly of n point charges is,

$$F_1 = \sum_{i=2}^{n} F_{1i} = F_{12} + F_{13} + F_{14} + ::: + F_{1n} : \qquad (1.5)$$

This is a mathematical representation of the principle of superposition ap-plied to electric forces (though not valid for very strong electric forces). Coulomb's law correctly describes the following concepts,

the electrical forces that bind the electrons of an atom to its nucleus,

the forces that bind atoms together to form molecules, and,

the forces that binds atoms and molecules to form solids or liquids.

Example 1: Consider three point charges q_1 = 1:2 C, q_2 = +3:7 C and q_3 = 2:3 C separated by distances r_{12} = 15 cm and r_{13} = 10 cm as illustrated on Figure 1.3. The charge q_3 makes an angle = 32^0 with the y-axis. The Coulomb force experienced by the charge q_1 due to the charge q_2 has a magnitude given by,

$$F_{12} = k \frac{q_1 q_2}{r_{12}^2}$$

$$= (8:99\ 10^9 Nm^2{=}C^2) \frac{(1:2\ 10\ _6C)(3:7\ 10\ _6C)}{(0:15m)^2} = 1:77\ N$$

The charges q_1 and q_2 have opposite signs, hence F_{12} is attractive force.

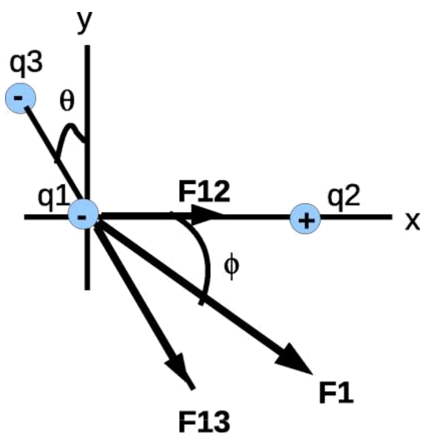

Figure 1.3: Three point charges on a Cartesian coordinate system.

Similarly, the Coulomb force experienced by the charge q_1 due to the charge q_3 has a magnitude is given by,

$$F_{13} = k \frac{q_1 q_3}{r_{13}{}^2}$$

$$= \ (8{:}99\ 10^9 Nm^2 = C^2) \frac{(1{:}2\ 10\ _6C)(2{:}3\ 10\ _6C)}{(0{:}1m)^2} = 2{:}48\ N$$

The charges q_1 and q_3 have the same signs, hence F_{13} is repulsive force. The components of the resultant force F_1 is calculated as follows,

$$F_{1x} \ = F_{12x} + F_{13x} \ = F_{12} + F_{13} \sin$$

$$= 1{:}77N + (2{:}48N)(\sin 32^0) = 3{:}08\ N$$

$$F_{1y} = F_{12y} + F_{13y} = 0 \qquad F_{13} \cos$$

$$= \ (2{:}48N)(\cos 32^0) = \quad 2{:}10\ N$$

so that in vector form, the resultant force F_1 is,

$$F_1 = F_{1x}\ \hat{}i + F_{1y}\ \hat{}j = \quad 3{:}08\hat{}i \quad 2{:}10\ \hat{}j\ N :$$

The magnitude of the resultant force F_1 is calculated as follows,

$$F_1 = \sqrt{(F_{1x})^2 + (F_{1y})^2} = 3{:}73\ \mathrm{N}$$

and the angle it makes with the x-axis is given by,

$$= \tan^{-1}\frac{F_{1y}}{F_{1x}} = 34{:}29^0 :$$

Properties of the electric charge

1. Quantisation: Just as matter is discrete, i.e. solids, liquids and gases are made up of atoms and molecules, the electric charge is also discrete or quantised. It's magnitude is in multiples of the elementary charge e. That means, any charge q that can be observed and measured directly, can be written as follows,

$$q = ne \qquad n = 0; \quad 1; \quad 2; \quad 3; ::: \qquad (1.6)$$

   in which e, the unit of the elementary charge has the experimentally determined magnitude of $e ’ 1{:}602\ 10^{19}$ C with an experimental uncertainty of about 3 parts in 10^7. Therefore the electrical charges for the electron, proton and neutron can be expressed as follows,

particle	charge(e)
electron	-1
proton	+1
neutron	0

   Protons and neutrons are composite particles made up of quarks. Protons have two up-quarks and one down-quark, and the neutrons have two down-quarks and one up-quark. The up- and down-quarks are as-signed charge $+\frac{2}{3}$ e and $-\frac{1}{3}$ e respectively, thus accounting for the net charge +e on the proton and zero charge on the neutron. The subject of elementary particle physics is rather beyond the scope of this course.

2. Conservation: When a glass rod is rubbed with silk, positive charge appears on the rod. Measurement shows that a corresponding negative charge appears on the silk. This suggests that rubbing does not create charge but merely transfers it from one object to another, disturbing slightly the electrical neutrality of each.

In nature, physical processes occur in such a way that charge is con-served. Consider an electron (charge -e) and a positron (charge +e) are brought close to each other. They annihilate each other convert-ing all their rest energy into radiation, typically two oppsitely directed gamma-ray photons each of energy 511 keV,

$$e \ + e^{+} \ ! \ + \ : \qquad (1.7)$$

Photons carry no charge. The net charge is zero both before and after the event and therefore charge is conserved. Another example is the decay of a neutral - meson or pion into two gamma-ray photons,

$$^{0}!+: \qquad (1.8)$$

The net charge is zero before and after the decay process. Next consider the decay of a neutron into a proton with the emission of a particle (fast moving electron) and an electron anti-neutrino $_e$,

$$n \ ! \ p + e \ + \ _e : \qquad (1.9)$$

The net charge is zero before and after the decay. The emission of an electron anti-neutrino is required to conserve electron lepton number. A detailed treatment of conservation laws in elementary particle physics is rather beyond the scope of this course.

The Electric Field: The electric eld E associated with a collection of charges is de ned in terms of the force F exerted on a positive test charge q_0 at a particular point or,

$$E = \frac{F}{q_0} \qquad (1.10)$$

The direction of the electric eld E is the same as that of the force F because q_0 is a positive scalar quantity. The SI unit for the electric eld is Newton per Coulomb (N/C).

Example 2: Consider a 5 nC test charge placed at a point such that it experiences a force of 2 10 ^{4} N in the x-direction. The electric eld at that point is,

$$E = \frac{2 \ 10^4 N}{5 \ \ 10^{9} C} \, i = 4 \ \ 10^4 \, i \ N{=}C$$

The electric eld of point charges: Let a positive test charge q_0 be placed at a distance r from a point charge q. The magnitude of the force acting on q_0 is given by Coulomb's law,

$$F = \frac{1}{4 \ _0} \frac{qq_0}{r^2} :$$

The magnitude of the electric eld at the site of the test charge is,

$$E = \frac{F}{q_0} = \frac{1}{4 \ _0} \frac{q}{r^2} :$$

To nd the resultant electric eld E on a point charge due to other point charges, the procedure is as follows;

calculate the electric eld E due to each charge at the given point as if it were the only charge present,

add the electric elds vectorially to nd the resultant eld, i.e.

$$E = E_1 + E_2 + E_3 + ::: = \sum_i E_i \qquad (i = 1; 2; 3; :::)$$

where,

$$E_i = \frac{1}{4 \ _0} \frac{q_i}{r_i^2} :$$

The sum is a vector sum taken over all the charges. This is also an exam-ple of the principle of superposition, which states in this context, that at a given point the electric elds due to separate charge distributions simply add up vectorially or superpose independently. This principle may fail when the magnitudes of the elds are extremely large, but it will be valid in all the situations discussed in this course.

Example 3: Consider the charges q_1 = 1:5 C and q_2 = 2:3 C placed on the x-axis as illustrated on Figure 1.4. The charge q_1 is placed at the origin and the charge q_2 is placed at a distance l = 13 cm. Since the two charges have the same sign, they will repel each other. At some point P along the x-axis, the electric elds strengths are equal and hence the resultant eld is zero.

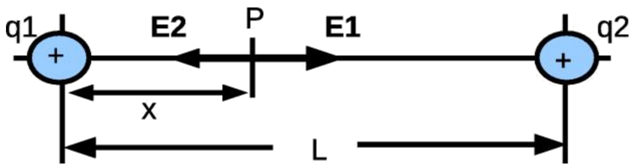

Figure 1.4: Two point charges along the x-axis on the Cartesian coordinate system. At the point P the two elds are equal and hence the resultant eld is zero.

If E_1 and E_2 are the electric elds due to the charges q_1 and q_2 respectively, then at the point P the two vectors must be equal, i.e. $E_1 = E_2$,

$$\frac{1}{4\;{}_0}\frac{q_1}{x^2} = \frac{1}{4\;{}_0}\frac{q_1}{(l \quad x)^2}$$

where x is the coordinate of the point P as illustrated on Figure 1.4. At the point P, the electric elds of the charges q_1 and q_2 are equal and opposite, so the net eld is zero. Solving the above equation for x yields,

$$x = \frac{l}{\quad q_2} = \frac{13\text{cm}}{\quad 2{:}3\ C} = 5{:}8\ \text{cm}$$

The solution x = 5:8 cm is a positive value and is less than l (= 13 cm), con rming that the zero- eld point P lies between the two charges.

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

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SPH101 Electricity and Magnetism I

Lecture No. 2.
Outline

- the lines of force for the electric eld
- the electric potential
- the electric dipole

The lines of force for an electric eld: It is often convenient to use lines in visualising the electric eld. They are usualy referred to as lines of force or electric eld lines and they have the following features,

- they indicate the direction of the electric eld, and,
- they originate on the positive charge(s) and terminate on the negative charge(s) as illustrated on Figure 1.5.

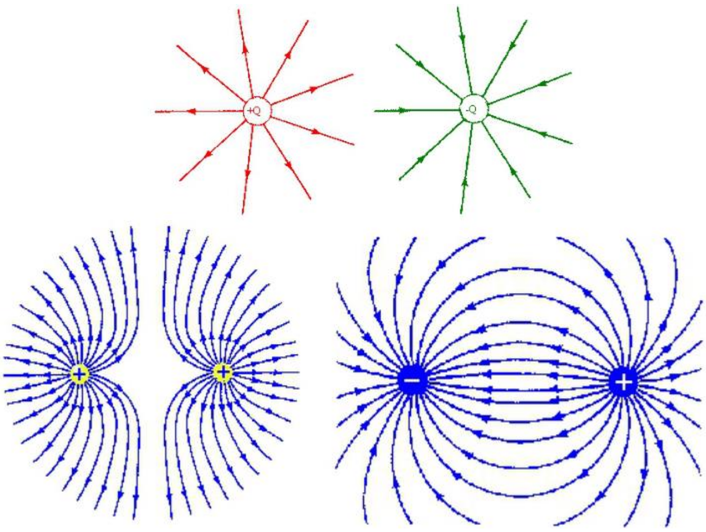


Figure 1.5: Illustrations of the electric eld lines.

The lines of force are drawn such that the number of lines per unit cross-sectional area (perpendicular to the lines) is proportional to the magnitude of the eld.

The electric potential: Consider a particle of charge q moving in a uniform electric eld E from an initial point a to a nal point b as illustrated on Figure 1.6. The particle experiences a force F given by,

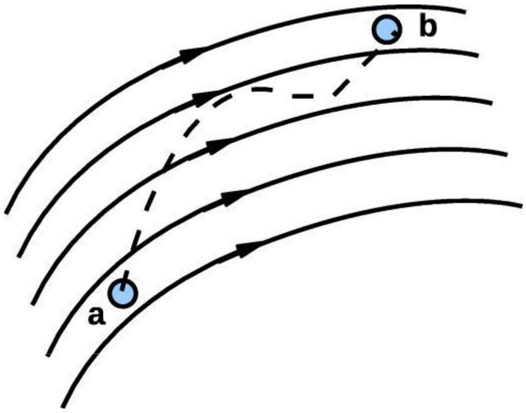


Figure 1.6: The motion of a point charge in a uniform electric eld E.

$$F = qE \tag{1.11}$$

often referred to as coulombic or electrostatic force. A potential energy is associated with any system in which a cahrged particle(s) is placed in an electric eld and acted on by an electrostatic force. The change in electrostatic energy, when a particle of charge q moves in an electric eld E is given by,

$$U_b \quad U_a = \int_a^b F\, ds = \quad q \int_a^b E\, ds : \tag{1.12}$$

U_a and U_b are the potential energies of the particle at points a and b. The integral is carried out over the path of the particle from initial point a to nal point b. The electric eld E (and hence force F) is conservative, the inte-gral is independent of the path. It depends only on the initial and nal point.

System of point charges: Consider a system of two point charges q_1 and q_2 as illustrated on Figure 1.7. The charge q_2 moves relative to the charge q_1 through a displacement dS. The electric eld E is due to the positive charge q_1.

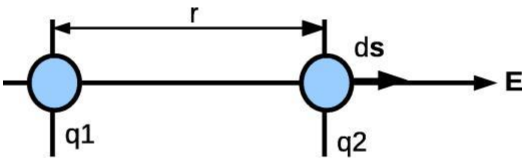


Figure 1.7: The motion of a point charge in a uniform electric eld E.

The change in potential energy is,

$$U = U_b - U_a = q_2 \int_{r_a}^{r_b} E_x \, dx$$

$$= \frac{1}{4\pi\epsilon_0} q_1 q_2 \int_{r_a}^{r_b} \frac{dr}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} q_1 q_2 \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$

If a reference point a is chosen such that r_a corresponds to an in nite sepa-ration of the particles, and we de ne the potential energy U_a to be zero. Let r be the separation at the nal point b, so that,

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \tag{1.13}$$

is an expression of the electrical potential energy of the system of point charges. Similarly, for a system of three point charges,

$$U(r) = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) : \tag{1.14}$$

The Potential Difference: As already seen, the change in potential energy q_2 moving from point a to point b is given by,

$$\Delta U = U_b - U_a = -q_2 \int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}\,.$$

The potential difference dV is defined as the potential energy change per unit charge,

$$dV = \frac{dU}{q_2} = -E\,dr$$

and for a finite displacement from point a to point b,

$$\Delta V = V_b - V_a = \frac{\Delta U}{q} = -\int_a^b \vec{E} \cdot d\vec{r}\,. \qquad (1.15)$$

The potential difference ΔV is the work per unit charge q necessary to move a test charge at constant speed from point a to point b. The SI unit for potential, the Joule per Coulomb, is called volt (V), i.e. 1 V = 1 J/C. Therefore the unit for the electric field E, the Newton per Coulomb, is also equal to volt per meter, i.e. 1 N/C = 1 V/m. A convenient unit of energy is the electron volt (eV), given by,

$$1\text{eV} = 1.6 \times 10^{-19}\text{C V} = 1.6 \times 10^{-19}\text{J}\,.$$

Example 1: Consider the Bohr model of the atom (electrons orbit round the nucleus in concentric circles). The first Bohr orbit in hydrogen atom has a radius $r \simeq 0.529 \times 10^{-10}$ m. The electric potentialbetween an electron and a proton is,

$$V = \frac{kq}{r} = \frac{(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2)(1.6 \times 10^{-19}\text{C})}{0.529 \times 10^{-10}\text{m}} = 27.19\ \text{V}$$

and the potential energy is,

$$U = qV = (-1.6 \times 10^{-19}\text{C})(27.19\text{V}) = -4.36 \times 10^{-18}\text{J} = -27.19\ \text{eV}$$

Example 2: Consider two protons that are 6.0 fm apart in the nucleus of a ^{238}U atom. The potential energy that associated with the electric force that acts between these two particles is given by,

$$U = k \frac{q_1 q_2}{r} = (8:99 \quad 10^9 \ Nm^2 = C^2) \frac{(1:6 \ \ 10^{19} C)^2}{6 \quad 10^{15} m}$$

$$= 3:84 \quad 10^{14} \ J = 240 \ keV$$

The two protons do not y apart because they are held together by the attractive strong force that binds nucleons in a nucleus together.

Example 3: An - particle (q = +2e) is is produced in a nuclear accelerator such that it moves from one terminal at a potential $V_a = 6:5 \ 10^6$ V to another terminal at zero potential, $V_b = 0$.

The corresponding change in the potential energy of the system is,

$$U = U_b \qquad U_a = q(V_b \qquad V_a)$$

$$= 2(1:6 \quad 10^{19} C)(0 \quad 6:5 \quad 10^6 V) = \quad 2:08 \quad 10^{12} \ J \ ' \quad 13:13 \ MeV$$

If no external force acts on the system, then it's mechanical energy,

$$E = U + K$$

remains constant. That is,

$$E = \qquad U + \qquad K = 0$$

hence,

$$K = \qquad U = 2:1 \quad 10^{12} \ J = 13:13 \ MeV$$

represents the gain in kinetic energy of the - particle.

Example 4: Consider a gold nucleus, with radius is 7 10 ^{15} m and the atomic number is 79. The nucleus is assumed to be spherically symmetric and behaves electrically for external points as if it were a point charge. The electric potential at the surface of the nucleus is then evaluated as follows,

$$V = k\frac{q}{r} = (8{:}99 \quad 10^{9}\ Nm^{2}{=}C^{2})\frac{(79)(1{:}6\ 10^{19}\ C)}{7\,10^{15}\ m} = 1{:}62 \quad 10^{7}\ V$$

This large positive potential has no e ect outside a gold atom because it is compensated by an equally large negative potential due to the 79 electrons in the atom.

The electric dipole: The gure 1.8 shows a positive and negative charge of equal magnitude placed a distance d apart. They constitute an electric dipole. The positive and negative charges set up electric elds E+ and E- respectively. The resultant electric eld at the point P is,

$$E{=}E{+}{+}E$$

The magnitudes of these two elds at point P are equal because P is equidis-tant from the positive and negative charges,

$$E{+}{=}E = \frac{1}{4\ \ _0}\frac{q}{r^2} = \frac{1}{4\ \ _0}\frac{q}{x^2 + (d{=}2)^2} :$$

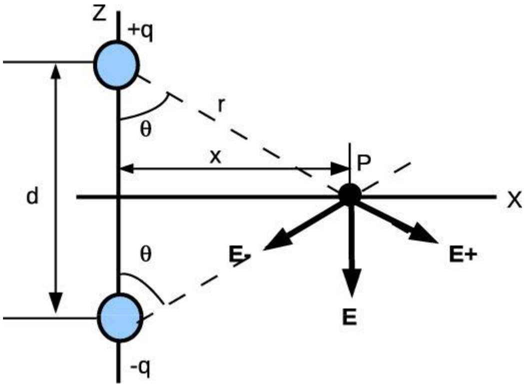


Figure 1.8: The electric dipole, made up of two opposite charges of magnitude q separated by distance d. At the point P on the x-axis, the resultant eld has only a z-component.

At the point P, the component of the resultant field along the x-axis is zero,

$$E_+ \sin\theta - E_- \sin\theta = 0$$

and along the z-axis,

$$E = E_+ \cos\theta + E_- \cos\theta = 2E_+ \cos\theta \, .$$

The angle θ is given by,

hence,
$$\cos\theta = \frac{d/2}{\sqrt{x^2 + (d/2)^2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{qd}{\left(x^2 + (d/2)^2\right)^{3/2}} \, . \qquad (1.16)$$

The total field is proportional to the product qd, called the electric dipole moment P, defined as,

$$P = qd \, . \qquad (1.17)$$

The dipole moment is a fundamental property of molecules, which often contain a negative charge and an equal positive charge separated by some definite distance.

Example 5: Consider the molecule of NaCl as composed of a Na^+ ion with a charge +e and a Cl^- ion with a charge -e. The separation between Na and Cl measured in NaCl is 0.236 nm. The dipole moment is expected to be,

$$P = ed = (1.6 \times 10^{-19} C)(0.236 \times 10^{-19} m) = 3.78 \times 10^{-29} C\,m$$

The measured value is 3×10^{-29} C m indicating that the electron is not entirely removed from Na and attached to Cl. To a certain extent, the electron is shared between Na and Cl, resulting in a dipole moment smaller than expected.

A dipole in an electric eld: Consider an electric dipole that consists of two equal and opposite charges +q and -q separated by a distance d. When it is placed in an external electric eld as illustrated on Figure 1.9, the force on the positive charge wii be in one direction and the force on the negative charge will be in another direction. To account for the net e ect of these forces, it is convenient to introduce the dipole moment vector P. The vector P has magnitude p = qd. and direction along the line joining the two charges pointing from the negative charge towards the positive charge.

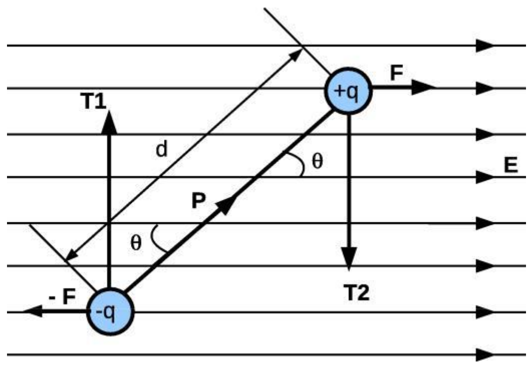


Figure 1.9: An electric dipole in an external eld E

The forces on the two charges are given as F = qE and opposite in direction, hence the net force is zero but there is a net torque T about its centre of mass that tends to rotate the dipole and bring the dipole moment vector P in alignment with the electric eld vector E. The magnitude of the torque is given by,

$$T = F\,(d=2)\sin\ + F\,(d=2)\sin\ = F\,d\sin \qquad (1.18)$$

with a direction perpendicular to the plane of the page and into the page, i.e.

$$T = (qE)d\sin\ = (qd)E\sin\ = P\,E\sin$$

or in vector form,

$$T = P\ E: \qquad (1.19)$$

The work done by the electric eld in turning the dipole from an initial angle $_0$ to a nal angle is,

$$W = \int dw = \int_{0} T\,d\ = \int_{0} T\,d \qquad (1.20)$$

where T is the torque exerted by the external eld. The vectors T and d are in opposite direction, so T d = T d .

$$W = \int_0 P E \sin d = P E \int_0 \sin d$$

$$= P E \cos \quad \cos_0 :$$

Since the work done by the agent that produces external eld is equal to the negative of the change in potential energy of the system of eld and dipole, wehave,

$$U = U( ) \quad U(_0) = \quad W = P E \cos \cos_0 :$$

$_0$ is arbitrarily de ned as the reference angle to be 90^0 and U( ) chosen to be zero at that angle. At any angle , the potential energy is then,

$$U = P E \cos$$

which can be written in vector form as,

$$U=PE \tag{1.21}$$

showing that the potential energy U is minimum when the dipole moment vector P and the electric eld E are parallel. The motion of a dipole in a uniform electric eld can therefore be intepreted either from the perspective of force or energy. The choice between the two is a matter of convenience in application to the problem under study.

Note that the resultant torque in the dipole tries to rotate it into alignment with the the direction of the external eld. The potential energy of the sys-tem tends to be a minimum when the dipole is aligned with the external eld.

Example 6: A molecule of water vapour (H_2O) has an electric dipole moment of magnitude $p = 6{:}2\ 10^{30}$ C m as illustrated on Figure 1.10. The dipole moment arises because the e ective centre of positive charge does not coincide with the e ective centre of the negative charge.

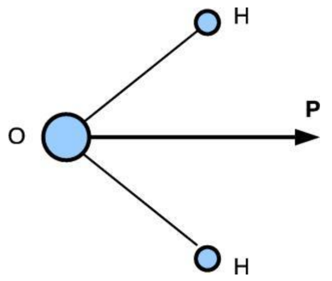


Figure 1.10: A molecule of water vapour with a dipole moment P.

(a) How far apart are the centres of positive and negative charges in a molecule of H_2O?

There are 10 electrons and correspondingly 10 positive charges in this molecule. The magnitude of the dipole moment is,

$$P = qd = (10e)d$$

from which the separation distance d can be evaluated,

$$d = \frac{P}{10e} = \frac{6{:}2\ 10^{30}\,\mathrm{C\ m}}{(10)(1{:}6\quad 10^{19}\mathrm{C})} = 3{:}88 \quad 10^{13}\ \mathrm{m}\ {:}$$

This is about 4% of the OH bond distance in this molecule.

(b) What is the maximum torque on a molecule of H_2O in a typical lab-oratory electric eld of magnitude $1{:}5\ 10^4$ N/C ?

The resultant torque is a maximum when $= 90^0$, hence,

$$T_{max} = P\,E\,\sin \quad = (6{:}2 \quad 10^{30}\ \mathrm{C\ m})(1{:}5 \quad 10^4\ \mathrm{N{=}C})(\sin 90^0) = 9{:}3 \quad 10^{26}\ \mathrm{N\ m}$$

(c) Suppose the dipole moment of a molecule of H_2O is initially pointing in a direction opposite to the eld. How much work is done by the electric eld in rotating the molecule into alignment with the eld?

The work done in rotating the dipole from = 180^0 to = 0^0 is given by,

$$W = P E \cos \quad \cos_0 = P E \cos 0^0 \quad \cos 180^0 = 2P E$$

$$= 2(6{:}2 \quad 10^{30} \text{ C m})(1{:}5 \quad 10^4 \text{ N=C}) = 1{:}86 \quad 10^{25} \text{ J}$$

By comparison, the average translational contribution to the initial energy (= 3/2 KT) of a molecule at room temperature is 6:2 10^{21} J, which is about 33,000 times larger than the calculated value of the work done to align the H_2O molecule. For the conditions of this problem, thermal agitation would overwhelm the tendency of the dipoles to align themselves within the eld. That is, if one had a collection of molecules at room temperature with randomly oriented dipoles, the application of an electric eld of this magnitude would have negligible e ect on aligning the dipole moments. This is because of the large internal energies. If one wishes to align the dipoles then much stronger elds and/or much lower temperatures must be used.

Assignment: Brie y describe the Millikan's oil drop experiment. Your description shoul include the following;

aim of the experiment

theretical framework

sketch of the experiment

observations and conclusions

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

SPH101 Electricity and Magnetism I

Lecture No. 3.
Outline

capacitors: parallel-plate, cylindrical,

spherical energy stored in capacitors

series and parallel arrangements of capacitors

Capacitors: As early as 1745, the german scientist Ewald Georg von Kleist found that a volume of water in a glass jar could store charges if some high voltage is applied to a wire in the jar. A year later, the danish physicist, Peter van Musschenbroek invented a similar device to store charges. This device, later named the Leyden jar[1] is an early form of a capacitor (also referred to as condenser).

The Leyden Jar is typically made of a glass jar with tin foil linnings and a brass rod terminating in an external knob at the top of the jar as illustrated on Figure 1.11. When an electrical charge is applied to the knob, positive and negative charges accumulate in the metal coatings of the glass jar. Leyden jars found useful applications to store electricity in experiments and also in some early wireless equipment.

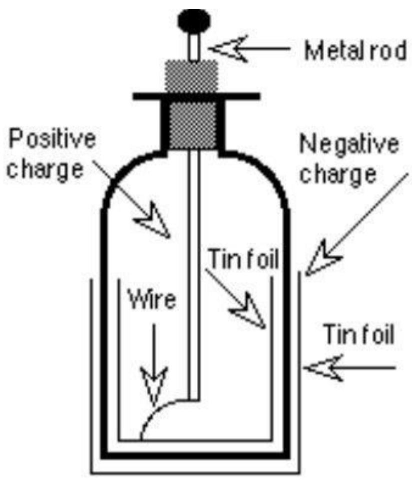


Figure 1.11: The sketch of a Leyden jar

[1]after the University of Leyden where Peter van Musschenbroek worked

Capacitors are devices that store electrical energy. Two conductors that are isolated from one another and their sorroundings form a capacitor. When a capacitor is charged, the conductors carry equal and opposite charges of magnitude q. The two conductors are called plates no matter what their shape is. Note that q is not the net charge on the capacitor. The net charge of the capacitor is zero. The magnitude of the charge on either plate of the capacitor will be denoted by q.

A capacitor can be charged by connecting the two plates to opposite termi-nals of a battery. In charging the capacitor, the battery transfers equal and opposite charges to the two plates. The potential di erence of the battery appears across the plates. The magnitude of the potential di erence is rep-resented by V and is directly proportional to the magnitude of the charge q on the capacitor, i.e.,

$$q \,/\, V \text{hence} \quad q = C\,V \tag{1.22}$$

in which C is the capacitance of the capacitor. This is a constant value and depends on the geometry of the capacitor. The SI unit for capacitance is the Farad (F),

$$1\text{ F} = 1\text{ Coulomb per Volt} = 1\text{ C=V}$$

named after Michael Faraday who, among his other contributions to science, developed the concept of capacitance.

Example 1: A storage capacitor on a random access memory (RAM) chip has a capacitance of 55 pF. If it charged to 5.3 V, how many excess electrons are there on it’s negative plate?

The charge on the plate of the capacitor is given by q = Ne and also q = CV , hence the number of excess electrons is given by,

$$N = \frac{q}{e} = \frac{CV}{e} = \frac{(55\ \ 10^{15}\text{ F})(5{:}3\text{ V})}{1{:}6\ \ 10^{19}\text{C}} = 1{:}8 \quad 10^{6}\text{ electrons:}$$

This is a very small number of electrons. A speck of household dust, so tiny that it essentially never settles, contains about 10^{17} electrons (and the same number of protons). Besides good aesthetics, this is one more reason to keep any form of dust away from electrical and electronic appliances.

Example 2: A 90 pF capacitor is connected to a 12 V and charged to 12 V. How many electrons are transferred from one plate to another?

The charge transferred is given by,

$$q = CV = (90 \quad 10^{12} F(12V) = 1{:}19 \quad 10^{19} C$$

so that the number of electrons transferred is,

$$N = \frac{q}{e} = \frac{1{:}19 \quad 10^{19}\ C}{1{:}6 \quad 10^{19} C} = 6{:}9 \quad 10^{9} \text{ electrons:}$$

The electric eld in a capacitor: The electric eld is related to the charge on the plates of a capacitor by Gauss' law,

$$_0 \oint E\, dA = q \tag{1.23}$$

or simply,

$$_0 E A = q$$

in which A represents the area of that part of the gaussian surface through which the ux (or electric eld lines) passes. The potential di erence between the plates is related to the electric eld E by,

$$V_f \quad V_i = \int_i^f E\, ds :$$

If the absolute value of the potential di erence is denoted as V then,

$$V = \int_+ E ds \tag{1.24}$$

in which the + and - signs indicate that the path of integeration starts on the positive plate and ends on the negative plate. The electric eld between the plates of a capacitor is the sum of the elds due to the plates.

Parallel-plate capacitor: Consider a parallel-plate capacitor illustrated on Figure 1.12. The plates of the capacitor are assumed to be su ciently large and close together so as to neglect the fringing of the electric eld at the edges of the plates.

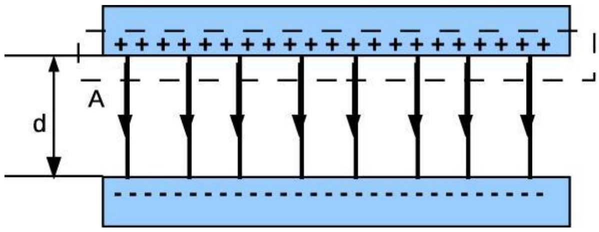


Figure 1.12: A cross-section of a parallel-plate capacitor. The Gaussian surface encloses the charge on the positive plate of the capacitor.

The magnitude of the electric eld is given by,

$$E = \frac{q}{{}_0 A} \qquad (1.25)$$

and the potential di erence is,

$$V = \int_{+} E ds = \frac{q}{{}_0 A} \int^{d} ds = \frac{qd}{{}_0 A} :$$

Since V = q=C then,

$$C = {}_0 \frac{A}{d} \qquad (1.26)$$

showing that the capacitance depends only on the geometrical factors namely plate area A and plate separation d.

Example 3: The plates of a parallel-plate capacitor are separated by a dis-tance d = 1 mm. What must be the plate area if the capacitance is to be 1 F?

$$A = \frac{C d}{{}_0} = \frac{(1 \text{ F})(1 \;\; 10^{3} \text{ m})}{8:85 \;\; 10^{12} \text{ F=m}} = 1:13 \;\; 10^{8} \text{ m}^{2}$$

Cylindrical capacitor: Consider a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b as illustrated on Figure 1.13. We assume that the length of the cylinders is much greater than their radii, L >> b so that we can neglect the fringing of the electric eld at the ends of the cylinders.

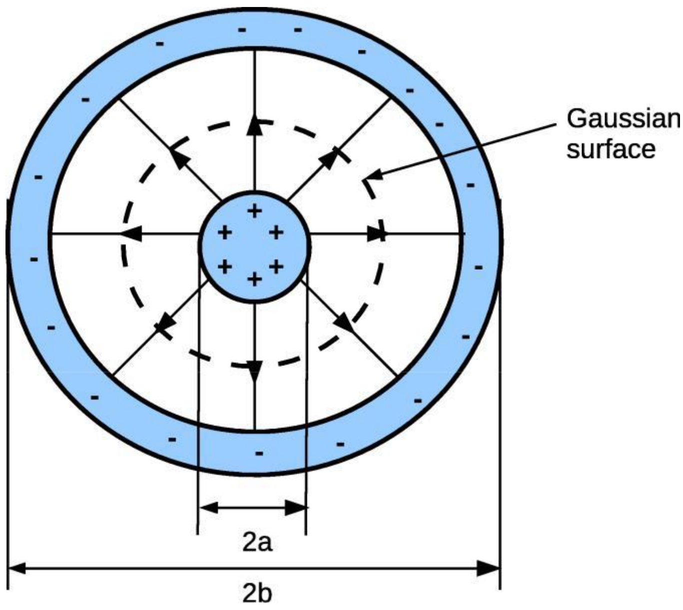


Figure 1.13: A cross-section of a cylindrical capacitor. The Gaussian surface encloses the charge on the positive inner cylinder of the capacitor.

As a Gaussian surface we choose a cylinder of length L and radius r closed by the end caps. The charge enclosed is,

$$q = {}_0 E A = {}_0 E (2 rL)$$

in which 2 rL is the area A of the curved part of the Gaussian surface.
The magnitude of the electric eld is therefore given by,

$$E = \frac{q}{2\ {}_0 Lr} \qquad (1.27)$$

so that the potential di erence across the plates is,

$$V = \int_{-}^{+} Eds = \frac{q}{2\ {}_0 L}\int_a^b \frac{dr}{r} = \frac{q}{2\ {}_0 L} \ln \frac{a}{b} :$$

The capacitance is therefore,

$$C=2\ _{0}\ \frac{L}{\ln(b=a)} \qquad (1.28)$$

which depends on the geometrical factors a; b; L.

Example 4: The space between the conductors of a long coaxial cable used in TV signal transmission has an inner radius a = 0:15 mm and an outer radius b = 2:1 mm. What is the capacitance per unit length of this cable?

The capacitance per unit length is given by,

$$\frac{C}{L} = \frac{2\ _{0}}{\ln(b=a)} = \frac{2\ (8{:}85\ \ 10^{\ 12}\ F{=}m)}{\ln(2{:}1{=}0{:}15)} = 21 \quad 10^{\ 12} F\ m:$$

Spherical capacitor: Consider a spherical capacitor which consists of two concentric spherical shells of radii a and b as illustrated on Figure 1.14. The charge enclosed is,

$$q = \ _{0} E A = \ _{0} E\ (4\ r^{2})$$

in which 4 r^2 is the area A of the spherical Gaussian surface.

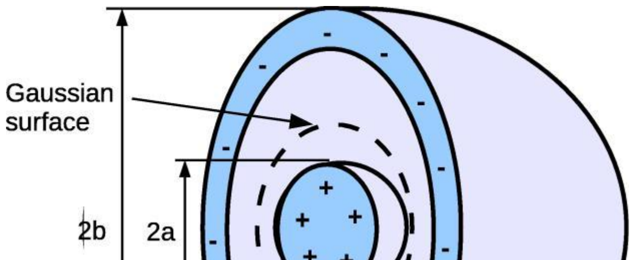


Figure 1.14: A cross-section of a spherical capacitor. The Gaussian surface encloses the charge on the positive inner sphere of the capacitor.

The magnitude of the electric field is therefore given by,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \qquad (1.29)$$

which is the expression for the electric field due to a uniform charge distribution. The potential difference across the plates is,

$$V = -\int_{+}^{-} E\,ds = \frac{q}{4\pi\epsilon_0}\int_a^b \frac{dr}{r^2}$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

The capacitance is therefore,

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}\,. \qquad (1.30)$$

Isolated sphere: We can assign capacitance to a single isolated conductor by assuming that the missing plate is a conducting sphere of infinite radius. After all, the field lines that leave the surface of a charged isolated conductor must end somewhere, the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius. If we let $b \to \infty$ and substitute R for a in Equation (1.30), then

$$C = 4\pi\epsilon_0 R \qquad (1.31)$$

the capacitance depends on the radius of the isolated spherical conductor.

Example 5: If the Earth is considered an isolated conducting sphere of radius 6370 km, then the capacitance is,

$$C = 4\pi\epsilon_0 R = 4\pi(8.85 \times 10^{-12}\ \mathrm{F/m})(6.37 \times 10^{6}\ \mathrm{km}) = 7.1 \times 10^{-4}\ \mathrm{F}\,.$$

Energy storage in an electric eld: Any charge con guration has a certain potential energy U, equal to the work W (which may be positive or negative) that is done by an external agent that assembles the charge con guration from their individual components which are assumed to be in nitely far apart and at rest. Consider a parallel capacitor: suppose at a time t some charge q has already been transferred from one plate to the other. The potential di erence V^{0} between the plated at that moment is $V^{0} = q^{0}=C$. If an increment of charge dq^{0} is now transferred, the resulting small charge dU in the electric potential energy is,

$$dU = V^{0}dq^{0} = \frac{q^{0}}{C}dq^{0} \tag{1.32}$$

where V^{0} and q^{0} denote instantaneous values of the potential di erence and charge respectively. If this process is continued untill a total charge q has been transferred, the total potential energy is,

$$U = \int dU = \int_{0}^{q} \frac{q^{0}}{C}dq^{0} = \frac{q^2}{2C} \tag{1.33}$$

and using q = CV then,

$$U = \frac{1}{2}CV^2 : \tag{1.34}$$

The energy stored in a capacitor resides in the electric eld between it's plates as we shall see in the next steps. In a parallel-plate capactitor, neglecting fringing e ects, the electric eld has the same value for all points between the plates. It follows that the energy density u, which is the stored energy per unit volume, should also be the same everywhere between the plates,

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad}$$

and on substituting the expression for capacitance in Equation (1.26),

$$u = \frac{{}_0}{2}\,V^2$$

However, v=d is the electric eld E, so that,

$$u = \frac{1}{2}\,{}_0E^2 : \tag{1.35}$$

This result, though derived for the special case of parallel-plate capacitor, hold for other capacitors in general. If an electric eld E exists at any point in space, then that point is a site for stored energy.

Capacitors with dielectric material: We have so far calculated the capacitance assuming that there is no material in the space between the plates of the capacitor. The region between the plates of a capacitor can be lled with a variety of insulating materials known as dielectrics. The presence of the dielectric materials alters the capacitance of the capacitors and (possibly) the electric eld between it's plates.

In 1837 Michael Faraday (1791 - 1867) investigated the e ect of lling the space between capacitor plates with dielectrics. He constructed two identical capacitors, lling one with dielectric and the other with air under normal conditions. When both capacitors were charged to the same potential difference, Faraday's epxeriments showed that the presence of the dielectric material increased the charge and hence the capacitance of a capacitor. The dimensionless factor by which the capacitance increases, relative to it's value C_0 when no dielectric material is present, is called the dielectric constant,

$$k_e = \frac{C}{C_0} \tag{1.36}$$

which is a fundamental property of the dielectric material and is independent of the size or shape of the conductor. Some typical dielectric materials are as follows.

Table 1.1: Some typical dielectric materials and their constants measured at room temperature.

material	dielectric constant k_e	dielectric strength (kV/mm)
vacuum	1 (exact)	1
air (1atm)	1.00059	3
polystyrene	2.6	24
paper	3.5	16
transformer oil	4.5	12
pyrex	4.7	14
mica	5.4	160
porcelain	6.5	4
silicon	12	
water (20^0C)	80.4	
titania ceramic	130	
strontium titanate	310	

Note that for most practical applications, air and vacuum are equivalent in their dielectric e ects. Every dielectric material has a characteristic dielectric strength, which is the maximum value of the electric eld it can tolerate without breakdown. The e ect of dielectric materials can be summarised as follows,

for a parallel plate capacitor lled with dielectric material of dielectric constant k_e, the capacitance is increased by a factor k_e,

$$C = k_{e\,0}\frac{A}{d} \quad ; \qquad (1.37)$$

for a point charge q embedded in a dielectric material, the electric eld is reduced by a factor k_e,

$$E = \frac{1}{4\,k_{e\,0}}\frac{q}{r^2} \quad : \qquad (1.38)$$

Generally, the presence of dielectric materials increase the charge stored (and hence capacitance) but weaken the electric elds (and hence the en-ergy stored) in capacitors.

Types of capacitors: Capactors may be divided into the following ve main groups according to the nature of the dielectric material used.

1. air capacitors: usually consist of one set of xed plates and another set of movable plates. They are mainly used in radio receivers where it is required to vary the capacitance.

2. paper capacitors: have electrodes which consist of metal foils inter-leaved with paper with waxor oil and rolled into a compact form.

3. mica capacitors: consist of alternate layers of mica and metal foils clamped tightly together or thin lms of silver sputtered on the two sided of mica substrate sheet.

4. ceramic capacitors: have electrodes which consist of metallic coat-ings (usually silver) on the opposite faces of a thin disc or plate of ceramic material such as silicate of magnesia or talc.

5. electrolytic capacitors: consist of two aluminium foils one with an oxide lm and another without, and interleaved with a material such as paper saturated with suitable electrolyte, e.g. ammonium borate.

Example 6: A parallel plate capacitor whose capacitance C_0 = 13:5 pF has a potential di erecne V = 12:5 V between it's plates. The charging battery is now disconnected and a porcelain slab (k_e = 6:5) is slipped between the plates. What is the stored energy of the unit, both before and after the slab is introduced?

The initial stored energy is,

$$U_i = \frac{1}{2} C_0 V^2 = \frac{1}{2}(13{:}5 \quad 10^{12}\ F)(12{:}5V)^2 = 1{:}055 \quad 10^{9}\ J = 1055\ pJ :$$

$$U_f = \frac{q^2}{C}$$

because, from the conditions of the problem statement, the charge q remains constant as the slab is introduced. After the slab is in place, the capacitance increases to $k_e C_0$, so that,

$$U_f = \frac{q^2}{2k_e C_0} = \frac{U_i}{k_e} = \frac{1055\ pJ}{6{:}5} = 162\ pJ$$

The energy after the slab is introduced is smaller by a factor of $\frac{1}{k_e}$. The missing energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a force on the slab, and would do work on it, given by,

$$W = U_i \quad U_f = 1055 \quad 162 = 893\ pJ$$

If the slab were introduced with no restriction, and if there were no friction, the slab would oscillate back and forth between the plates. The system consisting of capacitor and slab has a constant energy of 1055 pJ. The energy is converted repeatedly between kinetic energy of the moving slab and stored energy of the electric eld.At the instant the oscillating slab lled the space between the plates, it's kinetic energy would be 893 pJ.

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

Chapter 2

R-C Circuits

SPH101 Electricity and Magnetism I

Lecture No. 4.
Outline

> Series and parallel arrangement of capacitors and
> resistors charging and discharging a capacitor through a
> resistor R-C circuits

Series and parallel arrangement of capacitors: Consider the arrangement of capacitors in Figure 2.1.

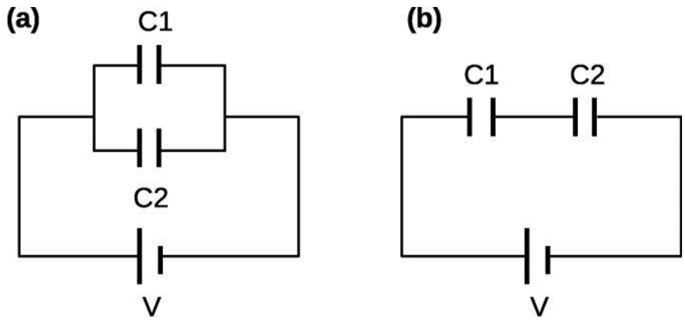


Figure 2.1: Arrangement of capacitors in (a) parallel and (b) series.

For the parallel arrangement of the capacitors in (a), $q_1 = C_1V$ and $q_2 = C_2V$, so that the total charge is,

$$q = q_1 + q_2 = CV$$

$$CV = C_1V + C_2V$$

so that the total or e ective capacitance is,

$$C = C_1 + C_2 \tag{2.1}$$

or in general for parallel arrangement of n number of capacitors, the e ective capacitance is given by a summation of all the capacitances,

$$C = \sum_{i=1}^{n} C_i \tag{2.2}$$

For the series arrangement of the capacitors in (b), $V_1 = \frac{q}{C_1}$ and $V_2 = \frac{q}{C_2}$, so that the voltage is,

$$V = V_1 + V_2 = \frac{q}{C}$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2}$$

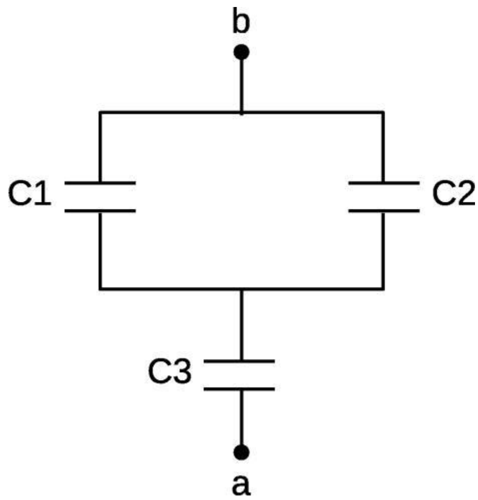
so that the total or e ective capacitance is,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \qquad (2.3)$$

or in general for series arrangement of n number of capacitors, the e ective capacitance is given by a summation of all the capacitances,

$$\frac{1}{C} = \sum_{i=1}^{n} \frac{1}{C_i} \qquad (2.4)$$

Example 1: Three capacitors C_1 = 12 F, C_2 = 5:3 F and C_3 = 4:5 F are connected as illustrated below.



Find the equivalent capacitance. A potential di erence of 12.5V is applied to the terminals a,b. Calculate the charge on the capacitor C_1.

The capacitors C_1 and C_2 are in parallel, so that their equivalent capacitance is,

$$C_{12}=C_1+C_2=12\ F+5{:}3\ F=17{:}3\ F$$

The capacitance C_{12} is in series with C_3, hence the equivalent capacitance is,

$$\frac{1}{C} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17{:}3} + \frac{1}{4{:}5} = 0{:}28(\ F)^{1}$$

so that,

$$C = \frac{1}{0{:}28} = 3{:}57\ F{:}$$

The total charge on the circuit is,

$$q = C\,V = (3{:}57\ F)(12{:}5\ V) = 44{:}6\ C$$

so that the voltate on the equivalent capacitance C_{12} is,

$$V_{12} = \frac{q}{C_{12}} = \frac{44{:}6\ C}{17{:}3\ F} = 2{:}68\ V$$

The same potential di erence appears on the capacitor C_1, hence the charge is,

$$q_1 = C_1\,V_1 = (12\ F)(2{:}68\ V) = 31\ C{:}$$

Resistors: Resistors are passive devices that provide electrical resistance to the ow of charges or current in electrical ciruits. The following are two broad categories of resistors.

1. linear resistors: also re erred to as ohmic conductors[1], the current through the resistor is directly proportional to the potential di erence across it,

$$V\ /\ I{:}$$

2. non-linear resistors: the electrical resistance varies, for example with temperature, in some semi-conductors or thermistors,

$$R = a\,e^{b=T}$$

   where a, b are constants. Some thermistors are prepared by embedding oxides of Mn, Fe, etc in ceramic binders and heating to high tempera-tures.

[1]they obey Ohm's law

Series and parallel arrangement of resistors: Consider the arrangement of resistors on Figure 2.2.

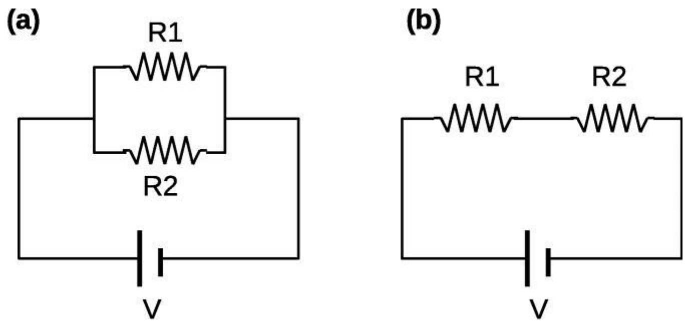


Figure 2.2: Arrangement of resistors in (a) parallel and (b) series.

For the parallel arrangement of the resistors in (a), currents I_1 and I_2 ow through resistors R_1 and R_2 such that the total current is,

$$I = I_1 + I_2$$

Since the potential di erence across the resistors has the same value V,

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}:$$

The total or e ective resistance is,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (2.5)$$

or in general for parallel arrangement of n number of resistors, the e ective resistance is given by,

$$\frac{1}{R} = \sum_{i=1}^{n} \frac{1}{R_i} \qquad (2.6)$$

For the series arrangement of the resistors in (b), the total potential di erence V is distributed in the resistors as follows,

$$V = V_1 + V_2$$

Since the current through the circuit has the same value I,

$$IR = IR_1 + IR_2:$$

The total or e ective resistance is,

$$R = R_1 + R_2 \qquad (2.7)$$

or in general for series arrangement of n number of resistors, the e ective resistance is given by a summation of all the resistances,

$$R = \sum_{i=1}^{n} R_i : \qquad (2.8)$$

Example 2: Three coils have resistances R_1 = 8 , R_2 = 12 and R_3 = 15 respectively. Calculate the equivalent resistance when they are connected in series and in parallel.

For the series arrangement,

$$R = R_1 + R_2 + R_3 = 8+12+15 = 35$$

For the parallel arrangement,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{8} + \frac{1}{12} + \frac{1}{15} = 0{:}275(\ )^{1}$$

hence,

$$R = \frac{1}{0{:}275} = 3{:}64 \quad :$$

Charging and discharging a capacitor: Consider an R-C circuit shown on Figure 2.3. Assuming that initialy there is no charge in the capacitor. When the switch is on position a the capacitor is gets charged.

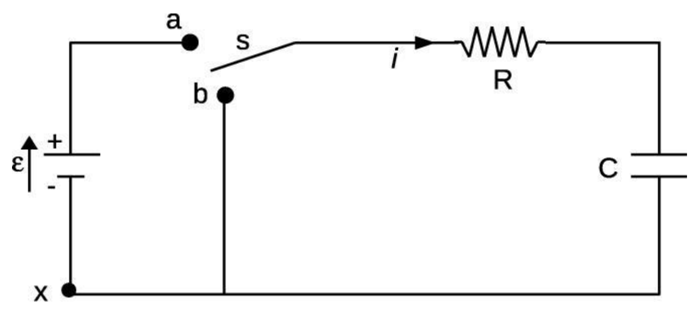


Figure 2.3: A simple R-C circuit for charging and discharging a capacitor.

In time dt acharge dq (= idt) moves through any cross-section of the circuit. The work ("dq) done by the seat of emf (electromotive force) must equal the internal energy (= i^2Rdt) produced in the resistor during the time dt plus the increase dU in the amount of energy stored in the capacitor. By conservation of energy,

$$\mathcal{E}dq = i^2Rdt + d\frac{q^2}{C}.$$

Since the capacitance is a constant,

$$\mathcal{E}dq = i^2Rdt + \frac{q}{C}dq \qquad (2.9)$$

which can be divided by dt to give,

$$\mathcal{E}\frac{dq}{dt} = i^2R + \frac{q}{C}\frac{dq}{dt}.$$

Noting that the current is equivalent to the rate of ow of charge with time, that is i = dq=dt,

$$\mathcal{E} = iR + \frac{q}{C}. \qquad (2.10)$$

This result is in agreement with the loop theorem - which is based on the principle of conservation of energy. Staring from point x on the circuit and going arround the circuit in a clockwise direction, we experience an increase in potential when going through the seat of emf and a decrease in emf when going through the resistor and capacitor,

$$\mathcal{E} - iR - \frac{q}{C} = 0 \qquad (2.11)$$

so that if we make the substitution i = dq=dt, and re-arrange the equation,

$$\frac{dq}{q - \mathcal{E}} = -\frac{dt}{RC}.$$

Integrating this result in the case that q = 0 at t = 0 one gets expressions for the charge and current at any time t,

$$q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) \qquad (2.12)$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}} \qquad (2.13)$$

and illustrated on Figure 2.4.

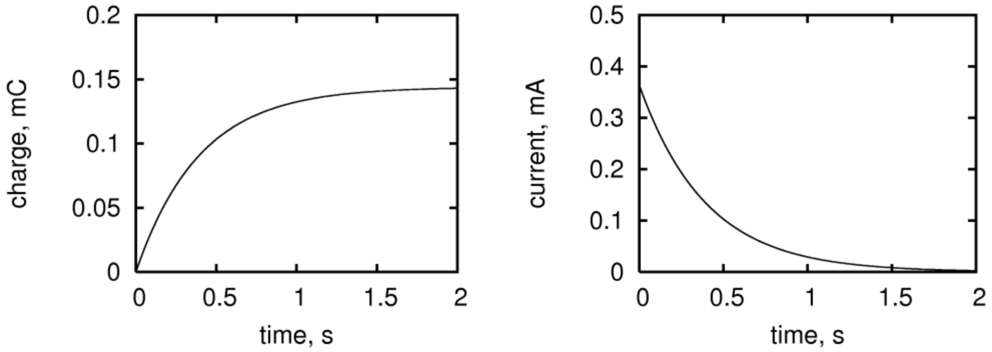


Figure 2.4: The charge and current when charging a capacitor. Note the values used C = 12 F, " = 12V, R = 33k .

At t = 0 the current through the circuit has a maximum value,

$$i = \frac{"}{R}$$

and then it decreases to zero, when the capacitor is fully charged,

q ' C" :

Initially there is no charge on the capacitor, hence zero potential di erence across it, and the current has a maximum value, hence a maximum potential di erence across the resistor. With time, the potential across the capacitor increases while that of the resistor decreases as illustrated on Figure 2.5.

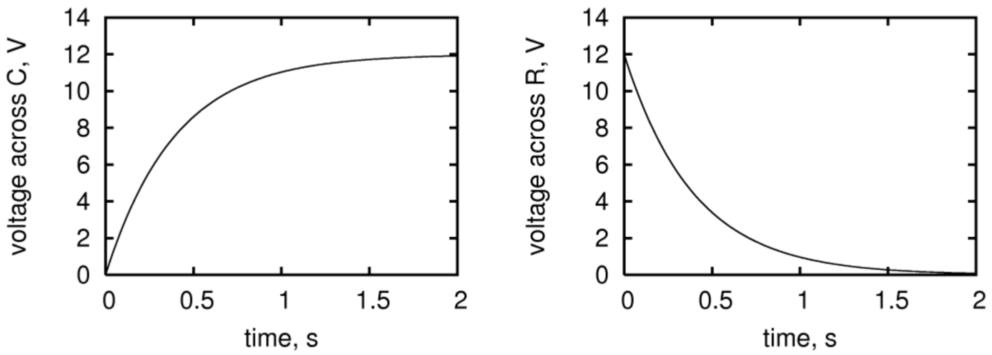


Figure 2.5: The voltage across capacitor and resistor when charging a capac-itor. Note the values used C = 12 F, " = 12V, R = 33k .

For the charging of a capacitor, the potential difference across the capacitor and resistor may be summarised as follows.

at $t = 0$ $V_R = \varepsilon$ $V_C = 0$,

as $t \to \infty$ $V_R \to 0$ $V_C \to \varepsilon$, and,

at all times $V_R + V_C = \varepsilon$.

The quantity RC has the dimensions of time and is called the capacitive time constant of the circuit,

$$\tau_C = RC. \tag{2.14}$$

It is the time at which the charge on the capacitor has increased to a factor of $(1 - e^{-1} \approx)$63% of it's final value, that is,

$$q = C\varepsilon\left(1 - e^{-1}\right) \approx 0.63\, C\varepsilon .$$

Assume now that the switch s has been in position a for a time that is much greater than RC. For all practical purposes, the capacitor is now fully charged and no current flows in the circuit. The switch is then thrown to position b so that the capacitor discharges through the resistor. There is no emf in the circuit, hence,

$$iR + \frac{q}{C} = 0 \tag{2.15}$$

or,

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

which can be re-arranged as follows,

$$\frac{dq}{q} = -\frac{dt}{RC}$$

whose solution is,

$$q(t) = q_0\, e^{-t/RC} \tag{2.16}$$

with $q_0 = \varepsilon C$ being the initial charge on the capacitor. The capacitive time constant RC appears in the expression for discharging a capacitor as well as in that for charging a capacitor. At a time $t = RC$ the capacitor charge is reduced to about 37% of the initial charge $(q_0 e^{-1})$.

The current during discharge is,

$$i = \frac{dq}{dt} = \frac{q_0}{RC} e^{t=RC} \quad (2.17)$$

with the negative sign indicating that the current is in the opposite direction, since the capacitor is discharging. Using $q_0 = C"$,

$$i = \frac{"}{R} e^{t=RC}$$

at t = 0, the initial current is,

$$i_0 = \frac{"}{R}:$$

as illustrated on Figure 2.6.

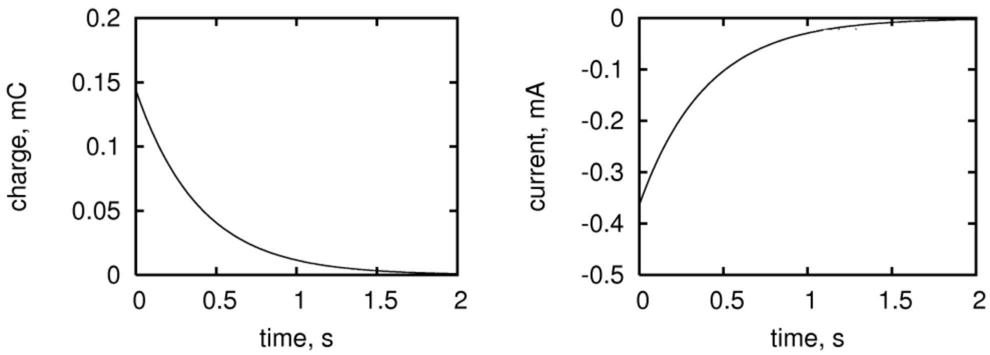


Figure 2.6: The charge and current when discharging a capacitor. Note the values used C = 12 F, " = 12V, R = 33k .

Initially there is maximum charge on the capacitor, hence maximum poten-tial di erence across it, and the current has a maximum value in opposite direction, hence a maximum negative potential di erence across the resistor. With time, the potential across the capacitor dereases while that of the resis-tor increases as illustrated on Figure 2.7. Note that the potential di erence across the capacitor falls exponentially from it's maximum value, which oc-curs at the time t = 0, where as the potential across the resistor is negative and rises exponentially to zero.

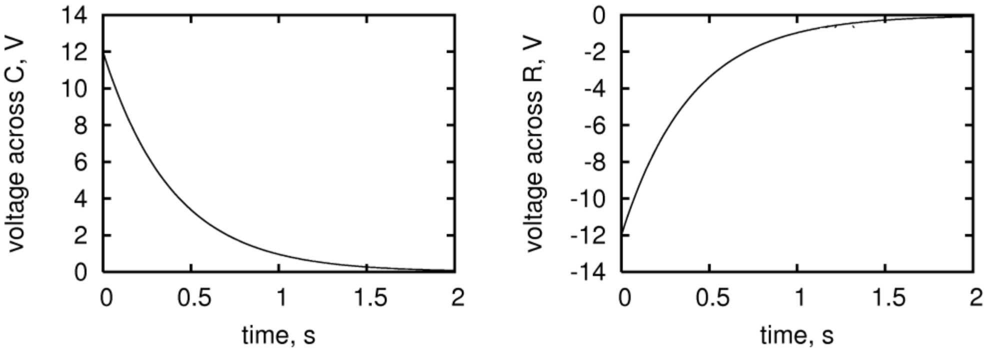


Figure 2.7: The voltage across capacitor and resistor when discharging a capacitor. Note the values used C = 12 F, " = 12V, R = 33k .

For the discharging of a capacitor, the potential di erence across the capac-itor and resistor may be summarised as follows.

at t = 0 V_R = " V_C = ",

as t ! 1 V_R ! 0 V_C ! 0, and,

at all times $V_R + V_C = 0$.

Example 3: A resistor (R = 6.2 M) and a capacitor (C = 2.4 F) are connected in series with a 12 V battery of negligible internal resistance.

(a) Calculate the capacitive time constant of the circuit.

The capacitive time constant of the circuit is,

$$_C = RC\ (6{:}2\quad 10^6\ )(2{:}4\quad 10\ \ 6\ F) = 15\ s :$$

(b) At what time after the battery is connected does the potential di erence across the capacitor equal 5.6 V?

The volatage across the capacitor is given by,

$$V_C = "1 \quad e^{\ t=RC}$$

which can be rearranged to calculate the time,

$$t = \ _C \ln \quad 1 \quad \frac{"_C}{V} \quad = \quad (15s) \ln \quad 1 \quad \frac{12}{5{:}6} \quad = 9{:}4\ s :$$

Example 4: Consider a capacitor C that discharges through a resistor R.

(a) After how many time constants does it's charge fall to half it's initial value?

(b) After how many time constants does the stored energy drop to half it's initial value?

At any time t the charge on the capacitor is given by,

$$q = q_0 e^{t=RC}$$

where q_0 is the initial charge. The time at which $q = q_0=2$ is determined as follows,

$$\frac{1}{2} q_0 = q_0 e^{t=RC}$$

which can be re-arranged,

$$t = (\ln 2)RC = 0{:}69RC = 0{:}69\ {}_{C}$$

the charge drops to half it's initial value after 0.69 time constants.

(b) The energy of the capacitor is,

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2C} e^{2t= C} = U_0 e^{2t= C}$$

in which U_0 is the initial stored energy. The time at which $U = U_0=2$ is determined as follows,

$$\frac{1}{2} U_0 = U_0 e^{2t= C}$$

which can be re-arranged,

$$t = {}_{C} \frac{\ln 2}{2} = 0{:}35\ {}_{C}$$

the stored energy drops to half it's initial value after 0.35 time constants have elapsed, that is faster than the loss of charge stored.

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

SPH101 Electricity and Magnetism I

Lecture No. 5.
Outline

- the electromotive force
- D-C circuits, Kirchho 's laws

The electromotive force: An energy source is required by most electrical circuits to move charges throughout the circuit. Devices that provide this energy are called sources of electromotive force or e.m.f.. Examples of sources of e.m.f. include ordinary battery, electric generator, solar cells etc. Figure 2.8 shows a seat of e.m.f. ", considered to be a battery, connected to a resistor R.

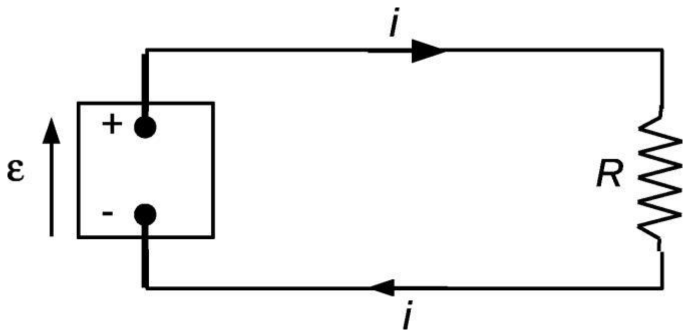


Figure 2.8: The seat of electromotive force (e.m.f.). A positive charge carrier would be driven in the direction shown by the arrows marked with i, a clockwise current is set up in the circuit. The actual direction of electrons in the opposite direction.

When a steady current has been established in the circuit, a charge dq passes through any cross section of the circuit in time dt. This charge enters the seat of e.m.f. " at it's low potential end and leaves at it's high potential end. The seat must do an amount of work dw on the (positive) charge carriers to force them to go to the point of higher potential. The electromotive force (e.m.f.) " of the seat is de ned as the work done per unit charge,

$$'' = \frac{dw}{dq} \quad : \qquad (2.18)$$

The unit for e.m.f. is the Volt, 1 Volt = 1 Joule=Coulomb .

The work done by a seat of e.m.f. on charge carriers in it's interior must be derived from a source of energy within the seat. The energy source may be, chemical (batteries), mechanical (generator), thermal, radiant (solar cells) etc. A seat of e.m.f. can therefore be described as a device in which some other form of energy is changed into electrical energy. The seat of e.m.f. is often represented by a symbol comprising two parallel lines, the larger one being the positive terminal as illustrated on Figure 2.9.

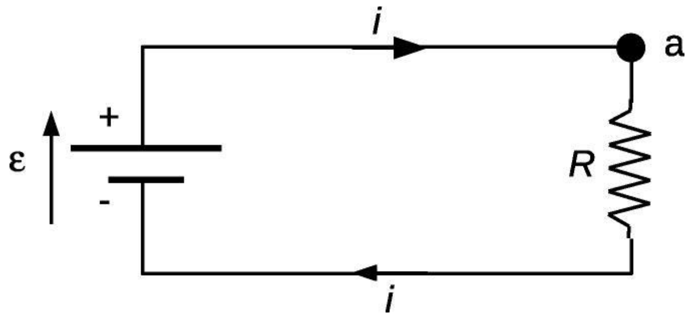


Figure 2.9: The seat of electromotive force (e.m.f.) in a simple D-C circuit

The work done by the seat of e.m.f. to move a charge dq (= idt) in time dt is,

$$dw = "dq = " i\, dt : \qquad (2.19)$$

An amount of energy i^2Rdt appears in the resistor as internal energy. From the principle of conservation of energy, the work done by the seat of e.m.f. must be equal to the internal energy deposited in the resistor,

$$"dq = " i\, dt = i^2 R\, dt \qquad (2.20)$$

or,

$$i = \frac{"}{R} : \qquad (2.21)$$

If we start at any point in the circuit and go round the circuit in either direction, adding up algebraically the changes in potential that we encounter, we must nd the same potential when we return to our starting point, that is,

the algebraic sum of the changes in potential enconuntered in a complete traversal of any closed circuit is zero.

This is the statement of Kirchho 's second rule (law) or simply the loop rule[2].

[2]formulated in 1845 by Gustav Robert Kirchho

Starting at point a on the circuit on Figure 2.9 and traverse the circuit in clockwise direction,

$$V_a \quad iR + " = V_a$$

$$iR + " = 0$$

which is independent of V_a and shows that the algebraic sum of the potential changes for a complete circuit is zero.

The following are some rules for nding the potential di erences,

1. if a resistor is traversed in the direction of the current, the change in potential is iR; and +iR for the opposite direction,

2. if a seat of e.m.f. is traversed in the direction of the e.m.f. (from the negative terminal to the positive terminal) the change in potential is +" and " in the opposite direction.

Internal resistance in a seat of e.m.f.: Oftenly, seats of e.m.f.have an internal resistance which is an inherent part of the device and cannot be removed (see Figure 2.10).

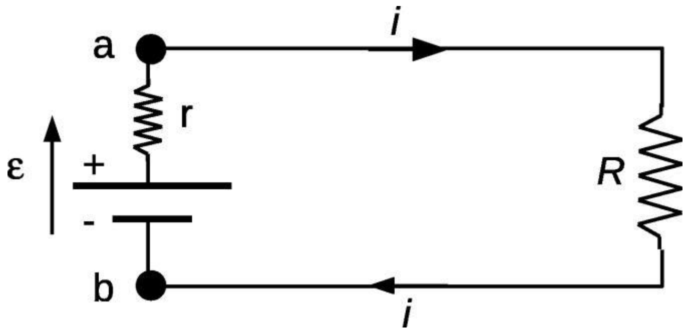


Figure 2.10: A seat of e.m.f. " with internal resistance r connected to a resistor R.

Applying the loop rule, starting from point b and going in the clockwise direction,

$$v_b + " \quad ir \qquad iR = V_b$$

so that the current is,

$$i = \frac{"}{R + r} : \qquad (2.22)$$

The results show that the internal resistance r reduces the current that the e.m.f.can supply to the external circuit. The potential di erence V_{ab} between the points a and b is,

$$V_{ab} = (V_a \quad V_b)$$

so that moving from the point b to a,

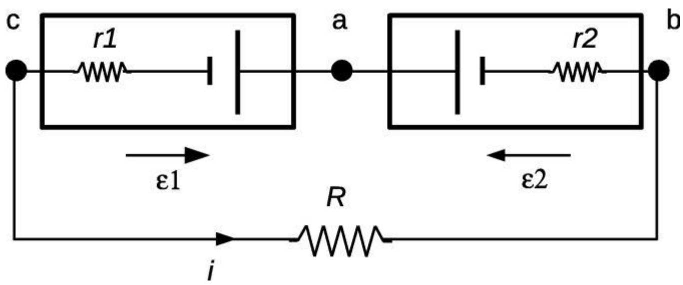
$$V_b + iR = V_a$$

hence,

$$V_{ab} = +iR = " \quad \frac{R}{R + r} \qquad (2.23)$$

which represents the potential di erence across the terminals of the seat of e.m.f. Note that V_{ab} = " if r = 0 (zero or negligible internal resistance) or if R = 1(external circuit is open).

Example 1: Consider two seats of e.m.f.'s $"_1$ and $"_2$ connected in series with a resistor R as illustrated in the following circuit.



The e.m.f.'s and the resistors have the following values:

$$"_1 = 2:1V; \; "_2 = 4:4V; \; r_1 = 1:8 \; ; \; r_2 = 2:3 \;\text{ and } R = 5:5$$

Calculate the following,

(a) the current through the circuit,

(b) the potential di erence between the points

(i) a and b

(ii) a and c.

The two e.m.f.'s are connected so that they oppose each other but $"_2$, because it is larger than $"_1$, controls the direction of the current in the circuit, which is counter-clockwise. The loop rule, applied clockwise from point a yields.

$$"_2 + i\, r_2 + i\, R + i\, r_1 + "_1 = 0$$

from which

$$i = \frac{"_2 \quad "_1}{R + r_1 + r_2} = \frac{4{:}4V \quad 2{:}1V}{5{:}5 \ + 1{:}8 \ + 2{:}3} = 0{:}24\ A$$

Starting from point b to point a in counterclockwise

$$\text{direction, } V_b \ i\, r_2 + "_2 = V_a$$

or

$$V_a V_b = \quad i\, r_2 + "_2 = \quad (0{:}24A)(2{:}3\ ) + 4{:}4V = 3{:}8V$$

Note that this is smaller than the e.m.f $"_2$ (4.4V) due to internal resistance r_2

If we try from b to a in clockwise direction.

$$V_b + iR + ir_1 + "_1 = V_a$$

or

$$V_b \quad V_a = i\, R + i\, r_1 + "_1 = (0{:}24A)(5{:}5\ \ + 1{:}8\ ) + 2{:}1 = 3{:}8V$$

we arrive to the same result.

Starting from point c to point a in a clockwise direction,

$$V_c\, i\, r_1 + "_1 = V_a$$

or

$$V_a V_c = \quad i\, r_1 + "_1 = \quad (0{:}24A)(1{:}8\ ) + 2{:}1V = 2{:}5V$$

This value is larger that the e.m.f $"_1$ (2.1V) due to $"_2$. If $"_1$ was a storage battery, it would be charging at the expense of $"_1$.

Multi-loop circuits: In practise, electrical and electronic circuits are composed of more than one loop, they are often multi-loop circuits. When multi-loop circuits are analysed, it is useful to consider their juctions and branches. A juction in a multi-loop circuit is a point in the circuit in which three or more wire segment meet as illustrated on Figure 2.11

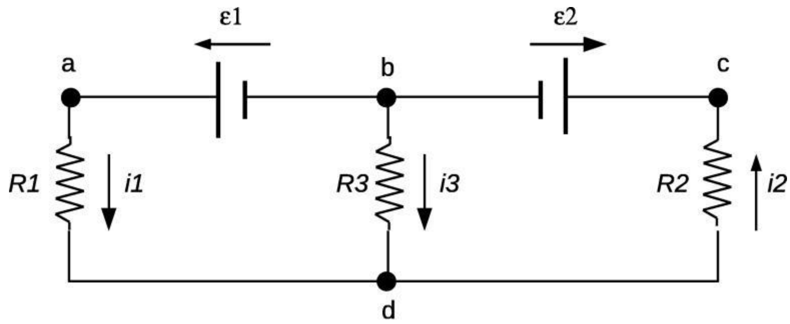


Figure 2.11: A two-loop circuits: Given the e.m.f's and resistances, one would like to nd the three currents i_1; i_2; and i_3 .

Note that there are 2 junctions b and d in the circuit. Points a and are not junctions because only two wire segments meet at those points. A branch is any circuit path that starts on one juction and proceeds along the circuit to the next junction. There are three branches in this circuit; three paths that connect the junctions b and d,

left branch b a d with current i_1

central branch b d with current i_2

right branch b c d with current i_3

In multi-loop circuit, each branch has it's own individual current, which can be determined through analysis of the circuit. Three current i_1; i_2 and i_3 have been shown in the circuit on Figure 2.11. The directions are chosen arbitrarily.

The three currents carry charge either toward junction d or away from it. Charge does not collect at juction because the circuit is in steady-state con-dition, charge must be removed from the junction at the same rate as that is brought into the juction, that is,

$$q_1 + q_3 = q_2$$

and since q = it therefore,

$$i_1 + i_3 = i_2 \quad (2.24)$$

At any junction the sum of currents leaving the juction equals the sum of currents entering the junction

This is known as Kirchho 's rst rule or junction rule. It is simply a statement of the conservation of charge.Therefore in the analysis of electric circuits there are two basic techniques (Kirchho 's rules);

1. conservation of charge - junction rule
2. conservation of energy - loop rule

If the two-loop circuit in Figure 2.11 is traversed in a counter clockwise direction starting from b and back to b, the loop rule gives,

$$\varepsilon_1 \quad i_1 R_1 + i_3 R_3 = 0 \quad (2.25)$$

The right loop gives (in counter clockwise direction)

$$i_3 R_3 \quad i_2 R_2 \quad \varepsilon_2 = 0 \quad (2.26)$$

The three simultaneous equations which can be solved for $i_1; i_2$ and i_3,

$$i_1 = \frac{\varepsilon_1(R_2 + R_3) \ \varepsilon_2 R_3}{R_1 R_2+R_2 R_3+R_1 R_3} \quad (2.27)$$

$$i_2 = \frac{\varepsilon_1 R_3 \ \ \varepsilon_2 (R_1 + R_3)}{R_1 R_2+R_2 R_3+R_1 R_3} \quad (2.28)$$

$$i_3 = \frac{\varepsilon_1 R_2 \ \ \varepsilon_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3} \quad (2.29)$$

showing that is in negative or in the opposite direction to that shown. To check these results, we can set $R_3 = 1$,

$$i_1 = i_2 = \frac{\varepsilon_1 \quad \varepsilon_2}{R_1 + R_2} \quad \text{and} \, i_3 = 0 :$$

The loop theorem can also be applied to the large loop consisting of the entire circuit a - b - c - d - a

$$i_1 R_1 \quad i_2 R_2 \quad "_2 + "_1 = 0$$

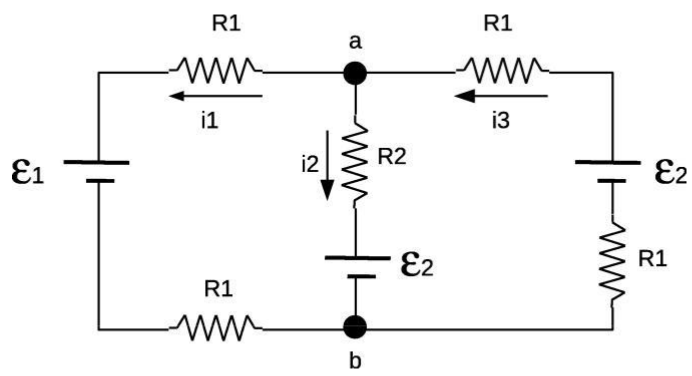
but is not an independent equation, it is simply the sum of (2.25) and (2.26).

For multi-loop circuits, the number of independent equations must equal the number of branches (or the number of di erent currents). The number of independent junction equations is one less than the number of junctions. The remaining equations must be loop equations.

Example 2: The gure below shows a circuit whose elements have the following values:

$$"_1 = 2{:}1\text{V} \qquad "_2 = 6{:}3\text{V} \qquad R_1 = 1{:}7 \qquad R_2 = 3{:}5$$

Find the currents in the three branches of the circuit, and the potential dif-ference between a and b.



The direction of currents are chosen arbitrarily. Applying the junction rule at a,

$$i_1 + i_2 = i_3 \tag{2.30}$$

starting at point a and traverse the left-hand loop in a counter clockwise direction,

$$i_1 R_1 \quad "_1 \quad i_1 R_1 + "_2 + i_2 R_2 = 0$$

or

$$"_2 \quad "_1 = 2\, i_1 R_1 \quad i_2 R_2 \tag{2.31}$$

Traversing the right-hand loop in a clockwise direction from point a

$$i_3 R_1 \quad "_2 + i_3 R_1 + "_2 + i_2 R_2 = 0$$

$$2\, i_3 R_1 + i_2 R_2 = 0 \qquad (2.32)$$

Equations 2.30, 2.31 and 2.32 are three independent simultaneous equations involving three variables i_1; i_2 and i_3. The solution of these equations leads to

$$i_1 = \frac{("_2 \quad "_1)(2 R_1 + R_2)}{4 R_1(R_1 + R_2)} = 0{:}82A$$

$$i_2 = \frac{"_2 \quad "_1}{2 (R_1 + R_2)} = 0{:}40A$$

$$i_3 = \frac{("_2 \quad "_1) R_2}{4 R_1 (R_1 + R_2)} = 0{:}42A$$

The result indicate that the directions of i_1 and i_2 are correct but the direction of i_3 is opposite to that shown. The potential di erence between points a and b, is obtained by traversing the branch a - b

$$V_a \quad i_2 R_2 \quad "_2 = V_b$$

$$V_a \quad V_b = "_2 + i_2 R_2$$

$$= 6{:}3V + (\quad 0{:}4A)(3{:}5\ ) = 4{:}9V$$

Showing that point a is at a higher potential than point b (as expected from the battery arrangements)

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

Chapter 3

Electrical Devices

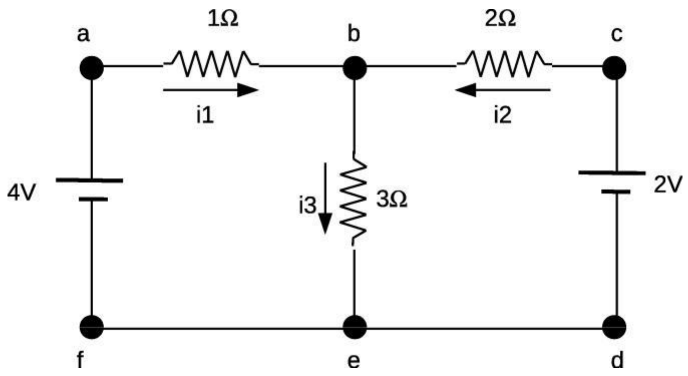
SPH101 Electricity and Magnetism I

Lecture No. 6.
Outline

the mesh current or loop current

method the Wheatstone bridge

Example 1: Consider the two-loop circuit below.



The currents i_1, i_2 and i_3 can be evaluated as follows. Applying Kirchho ’s junction rule at point b,

$$i_1 + i_2 \quad i_3 = 0 \qquad (3.1)$$

Applying Kirchho ’s loop rule, to the loop abef from point a traversing clockwise round the loop back to a,

$$V_a \; i_1 (1\ ) \; i_3 \; (3\ ) + 4V = V_a$$

or

$$i_1 (1\ ) + i_3 \; (3\ ) = 4V \qquad (3.2)$$

Similarly, traversing the right loop bcde in a clockwise direction about point b,

$$V_b + i_2 (2\ ) \quad 2\ V + i_3 (3\ ) = V_b$$

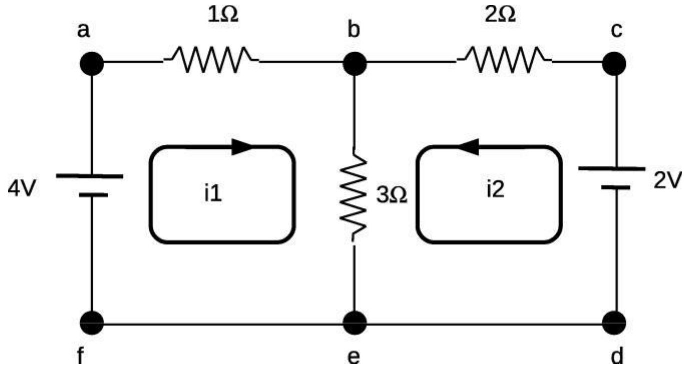
or

$$i_2 (2\ ) + i_3 (3\ ) = 2V \qquad (3.3)$$

Solving the three independent equations (3.1), (3.2)and (3.3) one gets.

$$i_1 = 1{:}27A \quad i_2 = \quad 0{:}36A \; i_3 = 0{:}91A$$

Mesh current or loop current method: Consider the two-loop circuit in the previous example. The number of unknowns (i_1; i_2; i_3) and the number of simultaneous equations can be reduced by assuming loop currents in the circuit.



The loop or mesh currents i_1 and i_2 are assumed to circulate in the loops a-b-e-f and b-c-d-e respectively. Applying Kircho 's voltage Law (loop rule) to the loop abef

$$i_1(1\ ) + (i_1 + i_2)(3\ ) = 4V \tag{3.4}$$

and to the loop cbde

$$i_2(2\ ) + (i_1 + i_2)(3\ ) = 2V \tag{3.5}$$

we have only two unknowns and two equations,

$$4\,i_1 + 3\,i_2 = 4V \tag{3.6}$$

$$3\,i_1 + 5\,i_2 = 2V \tag{3.7}$$

with the solutions,

$$i_1 = \frac{14}{11}A \quad \text{and} \quad i_2 = \frac{4}{11}A \quad \text{hence} \quad i_3 = i_1 + i_2 = \frac{10}{11}\ A:$$

The Wheatstone bridge: Consider the circuit on Figure 3.1 which consists of four resistors connected in a bridge network, known as the Wheatstone bridge.

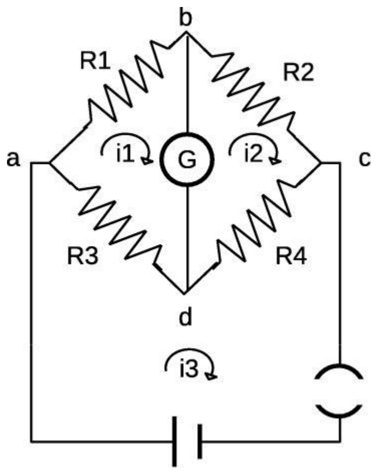


Figure 3.1: The Wheatstone bridge

Applying the voltage law to mesh or loop abd

$$R_1\, i_1 = (i_1 \quad i_2)\, R_g + (i_1 \quad i_3)\, R_3 = 0 \qquad (3.8)$$

For the loop bcd

$$R_2\, i_2 + (i_2 \quad i_3)\, R_4 + (i_2 \quad i_1)\, R_g = 0 \qquad (3.9)$$

If the network is balanced, the galvanometer current is zero. i.e i_1 I_2
hence equation 3.8 becomes,

$$R_1\, i_1 = (i_1 \quad i_3)\, R_3 = 0$$

or

$$(R_1 + R_3)\, i_1 = R_3\, i_3 \qquad (3.10)$$

Similarly, equation 3.9 becomes

$$R_2\, i_1 = (i_1 \quad i_3)\, R_4 = 0$$

or

$$(R_2 + R_4)\, i_1 = R_4\, i_3 \qquad (3.11)$$

dividing equation (3.10) by (3.11)

$$\frac{R_i + R_3}{R_2 + R_4} = \frac{R_3}{R_4}$$

cross multiplying and simplifying, to obtain,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

which is the condition for balance of the Wheatstone net or bridge. This is often used to determine unknown resistances. Figure 3.2 shows a simple form of a Wheatstone bridge.

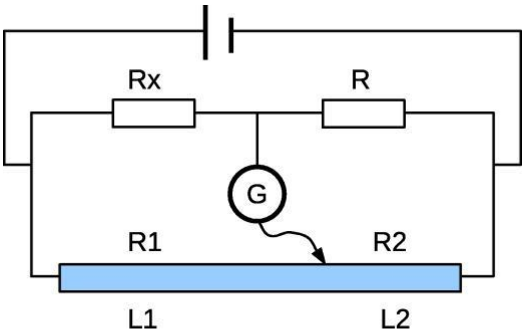


Figure 3.2: A typical circuit diagram for a Wheatstone bridge using a 1 m long slide wire.

When the bridge is balanced, i.e. there is no current owing in the galvanometer, the value of an unknown resistance R_X is evaluated as follows,

$$R_X = \frac{L_1}{L_2} R : \tag{3.12}$$

Example 2: A wheatstone bridge is composed of a xed resistor R = 200 and 1 m slide wire of uniform cross section. Calculate the value of the un-known resistance when the bridge balances at the 18 cm mark.

Given L_1 = 18 cm and L_2 = 100 18 = 82 cm, then,

$$R_X = \frac{L_1}{L_2} R = \frac{18}{82} (200\ ) = 43{:}9$$

The Potentiometer: is a device for measuring an unknown e.m.f $"_X$, by comparing it with a known standard e.m.f $"_S$. The basic elements of a potentiometer are shown on Figure 3.3

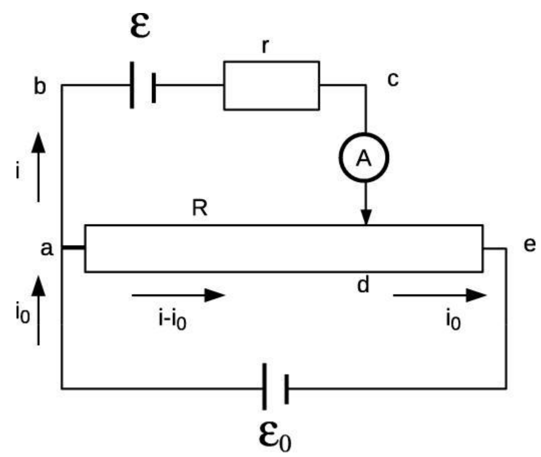


Figure 3.3: Schematic diagram of the Potentiometer

The resistor that extends from a to e is a carefully made precision resistor with a stiding contact shown at position d. The resistance R is the resistance between a and d. The source of standard emf $"_S$ is rst placed at " and the sliding contact is adjusted until the current is zero as noted by the sensitive ammeter A. The potentiometer is then said to be balanced, the value of R at balance being R_S. In this balance condition, we have, considering loop a-b-c-d-a

$$"_S = i_O R_S : \tag{3.13}$$

At the balance condition of the potentiometer, i = 0 hence the internal resistnce r of the standard source of e.m.f (or ammeter) has no e ect in our calculations. The process is repeated for an unknown e.m.f $"_X$ and the new balance condition is,

$$"_X = i_O R_X \tag{3.14}$$

On dividing equation (3.13) by (3.14) to get an expression for the unknown source of e.m.f.

$$"_X = "_S \frac{R_X}{R_S} : \tag{3.15}$$

Therefore, the unknown e.m.f can be determined interms of the known e.m.f by making adjustment on the precision resistor.

In it’s simplest form, the potentiometer consists of a wire MN of uniform cross-section, stretched alongside a scale and connected across a secondary cell B of ample capacity as illustrated on Figure 3.4.

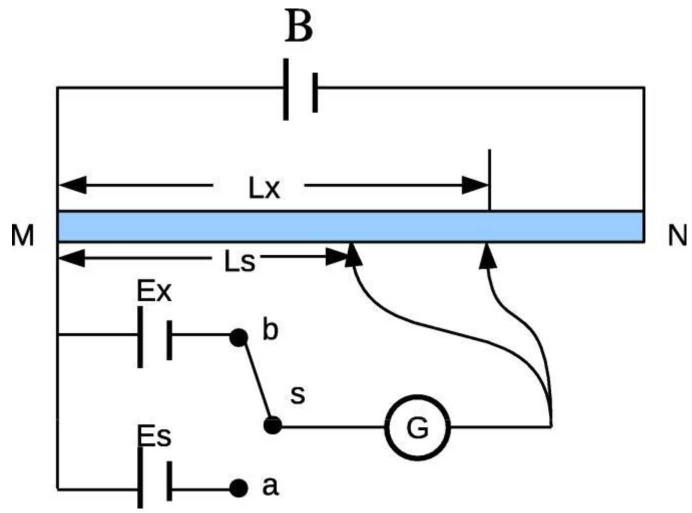


Figure 3.4: Typical circuit diagram of a potentiometer for measuring unknown e.m.f. The balance lengths are shown as L_S and L_X for the standard source of e.m.f. and the unkown e.m.f. respectively.

The resistance of a wire is given by

$$R = \frac{L}{A} \qquad (3.16)$$

where is the resistivity of the wire. Therefore the resistances R_S and R_X can be expressed in terms of the balance lengths L_S and L_X as follows,

$$R_S = \frac{L_S}{A} \; ; \quad R_X = \frac{L_X}{A} :$$

Equation 3.15 can therefore be written as follows,

$$"_X = "_S \frac{L_X}{L_S} \qquad (3.17)$$

showing that the unknown e.m.f can be determined from the balance points on the wire of uniform cross-section.

Example 3: A standard cell of e.m.f 1.01859V is balanced at 42cm along the uniform wire of a potentiometer. Determine the value of an unknown e.m.f balanced at 31cm along the wire of the same potentiometer.

Given $E_S = 1.01859V$, $l_S = 42cm$, $l_X = 31cm$.

$$E_X = 1{:}01859V \quad \frac{31}{42} = 0{:}752V$$

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

SPH101 Electricity and Magnetism I

Lecture No. 7.
Outline

the potential divider

the galvanometer

The Potential divider: is a device used to provide a variable potential difference from zero to the full supply value V_0 of a primary source as illustrated on Figure 3.5

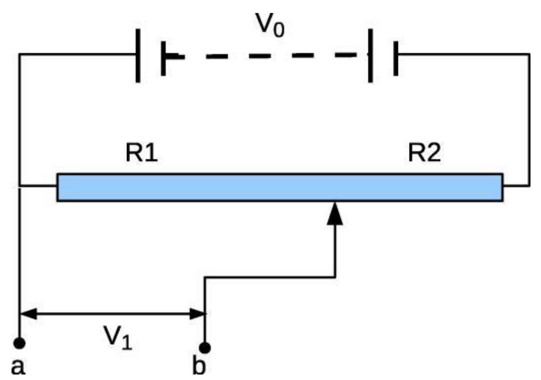


Figure 3.5: A typical circuit diagram for a potential divider.

In the absence of a load across the terminal a, b, the current I through the circuit is given by,

$$I = \frac{V_0}{R_1 + R_2}$$

and therefore

$$V_1 = IR_1 = \frac{R_1}{R_1 + R_2} V_0 \qquad (3.18)$$

which is a fraction of the full supply V_0

However, in the presence of a load, the load resistance R_3 will be in parallel with R_1 as illustrated on Figure 3.6. Equation 3.18 will therefore be no longer valid and the voltage V_1 must be measured with a voltmeter.

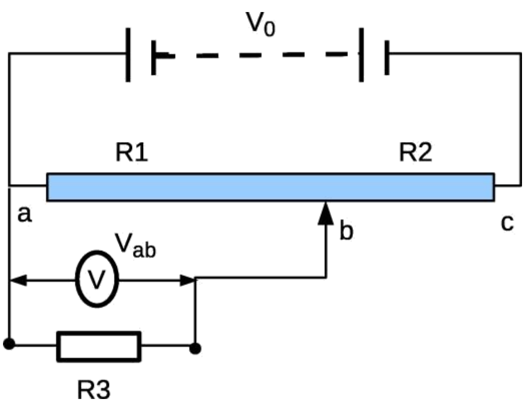


Figure 3.6: A typical circuit diagram for a potential divider with a load resistance.

If the load resistance R_3 is known then V_1 can be calculated as follows,

$$\frac{1}{R} = \frac{1}{R} + \frac{1}{R}$$

$$R_{ab} = \frac{R_1 R_3}{R_1 + R_3}$$

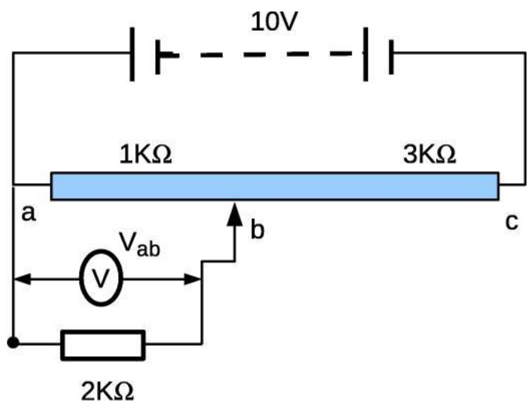
therefore

$$R_{ac} = R_{ab} + R_{bc} = \frac{R_1 R_3}{R_1 + R_3} + R_2$$

so that,

$$V_{ab} = \frac{R_{ab}}{R_{ac}} V_o :$$

Example 1: A load of 2k is connected via a potential divider of resistance 4k to a 10V supply. Calculate the potential di erence accross the load when the slider is one quarter up the divider?



$$R_{ab} = \frac{2000 \quad 1000}{2000 + 1000} = \frac{2000}{3} \qquad R_{ac} = 3000 + \frac{2000}{3} = \frac{11000}{3}$$

$$V_{ab} = \frac{R_{ab}}{R_{ac}} V_o = \frac{2}{11} \quad 10 = 1{:}8V$$

if the load is removed

$$V_{ab} = \frac{1000}{4000} \; 10V = 2{:}5V$$

Similarly, when the slider is half-way up the divider,

$$R_{ab} = \frac{2000 \quad 2000}{2000 + 2000} = 1000 \qquad R_{ac} = 1000 + 2000 = 3000$$

$$V_{ab} = \frac{1000}{3000} \; 10V = 3{:}3V$$

and if the load is removed,

$$V_{ab} = \frac{2000}{4000} \quad 10V = 5V$$

Galvanometer: The main components of a moving coil galvanometer are a set of permanent magnets enclosing a coil of wire free to turn with an at-tached pointer and a scale, as illustrated on Figure 3.7

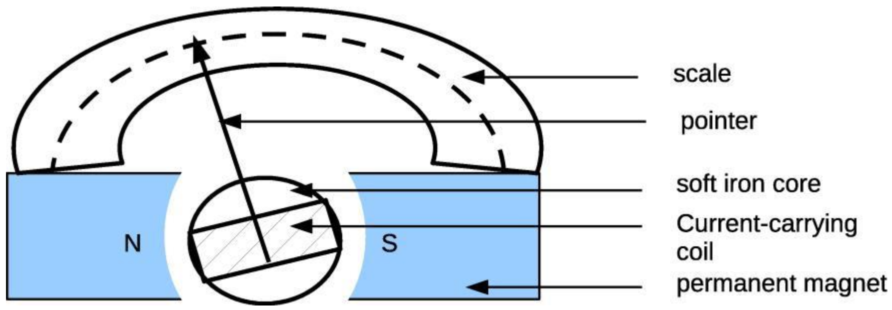


Figure 3.7: A typical moving coil galvanometer

The principle of operation is as follows. A coil carrying a current in a magnet eld experiences a torque which is proportional to the current I i.e

/ I

For a coil of N turns in a magnetic eld B the torque on the coil is

= N i A B Sin

where,

A is the area of the closed loop of the coil

is angle between the normal perpendicular to the plane of the loop and the magnetic eld B,

i is the current through the coil.

A rectangular loop of wire carrying a current i placed in a uniform magnetic eld. The unit vector n^ is normal to the plane of the loop, and makes an angle with the eld B . A torque acts to rotate the loop about the Z axis so that n^ aligns with B This torgue rotates the coil until it is balanced by the restoring torque provided by the mechanical suspension of the coil. Since the restoring torque of the suspension is proportional to the angle of the coil, the equilibrium angle of rotation will be proportional to the current in the coil.

Therefore, the resistance of the galvanometer and the current needed to produce full-scale de ection are the two parameters important for the construc-tion of an ammeter or voltmeter from a galvanometer. Typical values of these parameters for a portable pivoted-coil laboratory galvanometer are R_g = 20 and I_g = 0.5mA. The voltage drop across a galvanometer with these parameters is thus

$$I_g R_g = 10\ ^{2}V$$

for full-scale de ection.

Ammeter: To construct an ammeter from a galvanometer, a small resistance called shunt resistor R_S, is connected in parallel with the galvanometer as illustrated on Figure 3.8.

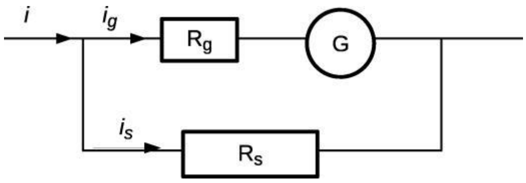


Figure 3.8: A typical circuit diagram for the conversion of an galvanometer into an ammeter. Note that the total current $i = i_g + i_S$.

The shunt resistor R_S is chosen such that $R_S << R_g$ so that most current ows into the shunt. The e ective resistance of ammeter R_A is much smaller than R_g ($R_A << R_g$)

$$R_A = \frac{R_s R_g}{R_S + R_g}$$

The shunt resistor is in parallel to the galvanometer, hence voltage across the shunt is the same as that across the galvanometer,

$$i_g R_g = i_S R_S$$

$$R_S = \frac{i_g}{i_S} R_g \qquad (3.19)$$

Hence an ammeter can be designed using an appropriate shunt resistance R_S for particular requirements of measured currents.

Example 2: Using a galvanometer with a resistance 20 , for which a current of 5 10 ^{4}A through it's coil, gives full-scale de ection, design an ammeter which will read full-scale when the current is 5A.

given

$$R_g = 20 \qquad i_g = 5 \;\; 10^{4}A \qquad i = 5A$$

$$i_S = i \qquad i_g \text{ ' } 5A$$

$$R_s = \frac{5\ 10^{4}A}{5A} \quad 20 = 2 \quad 10^{3}$$

i.e R_S<<R_g the resistance of the shunt is much smaller than the of the current must ow through the shunt resistor R_S and the e ective resistance of the parallel combination is approximately equal to that of the shunt.

$$R = \frac{R_S R_g}{R_s + R_g} = \frac{(20\ \ 10^{3})(20\ )}{(20\ + 20\ \ \ 10^{3})} = 20 \quad 10^{3}$$

Voltmeter: A resistor R_S is connected in series to a galvanometer in order to construct a voltmeter as illustrated on Figure 3.9

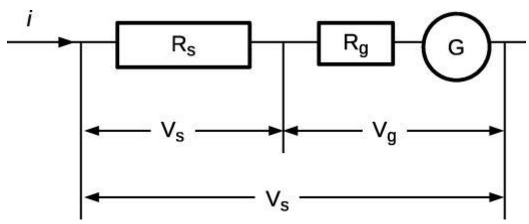


Figure 3.9: A typical circuit diagram for the conversion of an galvanometer into a voltmeter. Note that the same current ows through the circuit $i = i_g$.

The series resistor R_S is chosen such that R_S>>R_g so as to drop the current through the galvanometer. Since the resistor is in series with the galvanome-ter,

$$V = V_S + V_g = i(R_S + R_g)$$

the series resistance is given by,

$$R_S = \frac{V}{i} \quad R_g : \tag{3.20}$$

Example 3: Design a voltmeter to measure a maximum of 10V at full-scale de ection using a galvanometer of resistance R_g = 20 which gives full-scale de ection when a current of i_g = 0.5mA ows through it's coil.

Given

$$V = 10V \qquad i = i_g = 5 \;\; 10^{\;4}A \qquad R_g = 20$$

then

$$R = \frac{V}{i} \quad R_g = = \frac{10V}{5 \; 10^{4}A} \quad 20 = = 19;980 \;\; ' \; 20k$$

A resistor R_S = 20k connected in series with the galvanometer would meet the design requirements.

Ohmmeter: It consists of a source of e.m.f., a galvanometer and a resistor connected as illustrated on Figure 3.10.

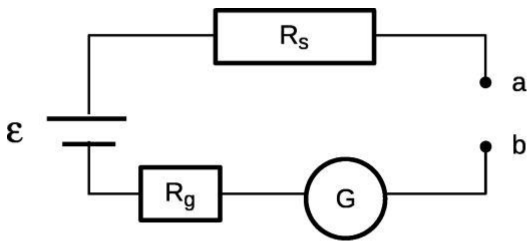


Figure 3.10: A typical circuit diagram for the conversion of an galvanometer into an Ohmmeter.

The resistance R_S is chosen to give full-scale de ection when the terminals a and b are shorted (connected together without any load resistance). The full-scale is marked zero-resistance. When the terminals are connected across an unknown resistance R, the current is less than i_g and the galvanometer reads less than full-scale. The current through the circuit will depend on the load resistance R,

$$I = \frac{"}{R + R_S + R_g} : \tag{3.21}$$

The galvanometer can be calibrated in terms of the resistance measured, from zero at full-scale to in nite resistance at zero de ection. The calibration is non-linear and depends on the constancy of the e.m.f of the battery. Hence such an ohmeter is NOT a high-precision instrument, though quite useful for making quick, rough determination of resistance.

In any case, some calibration must be excercised in the use of an ohmeter since it sends a current through the resistance to be measured. For example, consider an ohmeter with a 1.5V battery on a galvanometer. Similar to that in the previous examples, the series resistance needed is,

$$I_g (R_S + R_g) = 1{:}5V$$

$$R_S = \frac{1{:}5}{5\ 10^4} \qquad R_g = 3000\ \ 20 = 2980$$

Suppose we were to use the ohmmeter to measure the resistance of a more sensitive galavanometer which gives full-scale reading with a current of $10\ ^5$A and has a resistance of about 20 . When the terminals a and b are placed across this more sensitive galvanometer, the current will be just slightly less than $5 \times 10\ ^4$A because the total resistance is 3000 , which is just slightly more than 3000 . Such a current, about 50 times that needed tp produce full-scale de ection, would ruin the more sensitive galvanometer! Wheatstone bridges provide a more accurate method to determine unknown resistances.

Assignment: A moving coil galvanometer gives a full-scale de ection with 15mA and has a resistance of 5 . Design the following,

i) an ammeter to measure upto 1A,

ii) a voltmeter to measure upto 10V,

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

Lecture No. 8.
Outline

the cathode - ray oscilloscope and it's applications

The Cathode Ray Oscilloscope (CRO): The CRO is almost universally employed to display the waveforms of alternating voltages and currents and has very many applications in electrical testing. The basic form of their operation is illustrated on Figure 3.11.

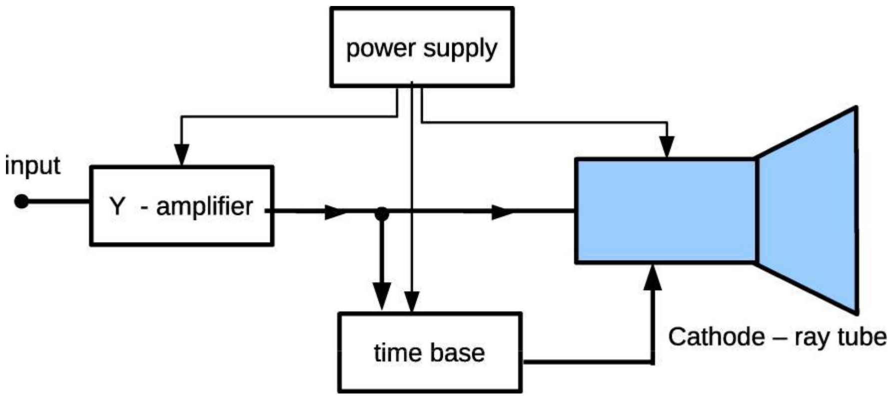


Figure 3.11: A schematic diagram of a CRO.

The cathode ray tube is an important component of the CRO. The principal features of the cathode ray tube are as follows,

cathode for production of electrons,

control grid with a variable negative bias to control the emission of electrons, thereby varying the brilliancy of the spot on the uorescent screen,

anode discs maintained at a high potential relative to the cathode, so that the electrons passing through the grid are accelerated very rapidly,

plates X and Y for the horizontal and vertical de ection of the electron beam,

as illustrated on Figure 3.12.

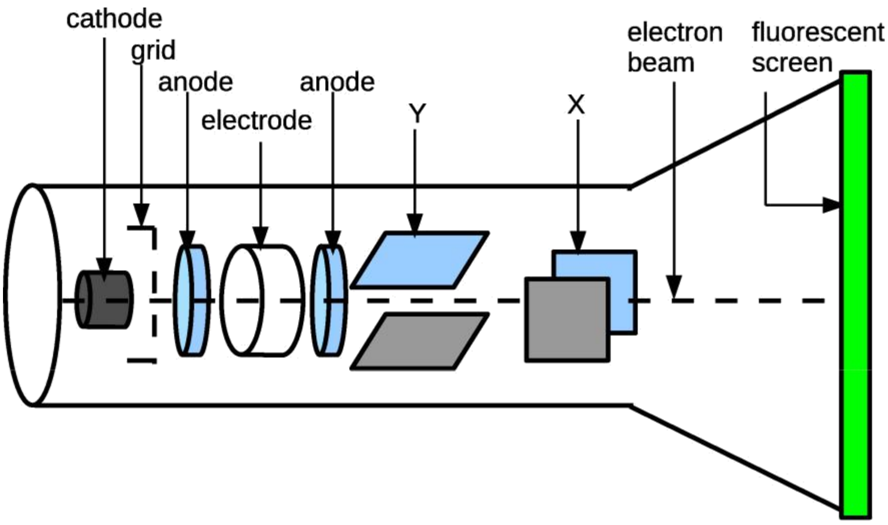


Figure 3.12: A schematic diagram of a cathode - ray tube.

The electrons shoot through the small appertures in the anode discs and them impact on the uorescent screen to produce a luminous patch or spot. This patch can be focussed into a bright spot by varying the potential of the focussing elecrode between the anode discs. This varyies the distribution of the electrostatic eld in the space between the anode discs. The electrode may consist of a metal cylinder or two discs with relatively large appertures. The combination of the anode discs and electrode may be regarded as an electron lens, and the system of electrodes producing the electron beam is termed as electron gun. The glass bulb housing the uorescent screen is evacuated to prevent any ionisation.

The electrons after emerging through the anode discs pass between two pairs of parrallel plates, termed the X- and Y- plates. One plate of each pair is usually connected to anode and the other is kept at ground potential. Sup-pose a d.c supply is supplied across the Y-plates. The eletrons constituting the beam will be attracted towards the positive plate and the beam will be de ected upwards. If the alternating voltage is applied across the Y-plates, the beam would therefore trace a vertical line on the screen. Similarly an alternating voltage applied to the X-plates would cause the beam to trace a horizontal line. The time-base serves to move the beam across the screen of the tube- achieved by the X-plates.

The power supply serves the following;

the grid and anode systems,

brilliance, focus and astigmatism controls,

and also to amplify the control system.

The electronic circuitry of the oscillope (details are beyond the scope of this course) are capable of handling a very wide range of input signals varying from a few millivolts to possibly a few hundred volts, while the input signal frequncy may vary from zero (d.c) upto possibly 1GHz, although an upper unit of 10-50MHZ is more common in general purpose instruments.

Measurement of voltage: An unknown a.c voltage is applied to the Y-plates of the CRO. If the time base is switched o , then a vertical trace is observed on the screen of the oscilloscope. The size of the vertical line trace is equivalent to the twice the amplitude or voltage of the unknown input signal. If the CRO is calibrated using known voltages then the value of an unknown voltage may be determined. If the time-base is switched on then the waveform of the signal may be displayed on the CRO screen as illustrated on Figure 3.13

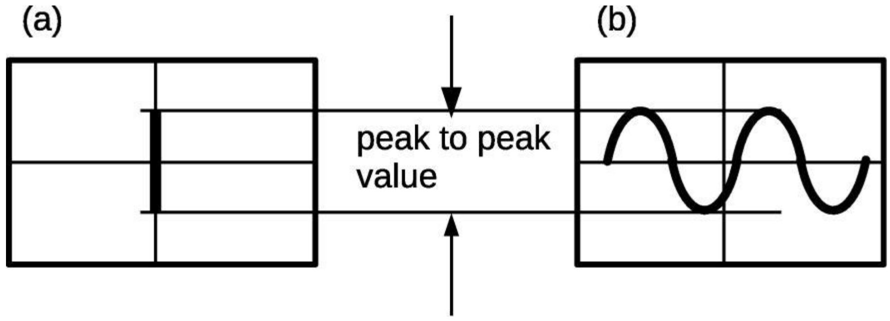


Figure 3.13: Traces of a signal on a CRO with the time base switched (a) o and (b) on.

Measurement of frequency: If a calibrated time-base is available, frequency measurements can be made. For example the trace of a square-wave input signal is displayed on a CRO as illustrated on Figure 3.14

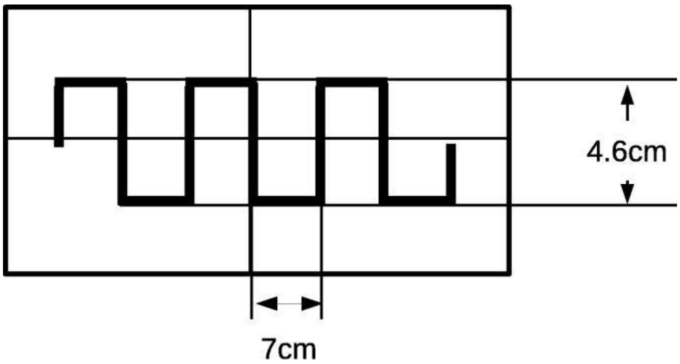


Figure 3.14: A typical square wave signal measured on a CRO.

The signal amplitude control is set to 0.5V/cm and the time-base control to 100 s/cm. The peak to peak voltage and frequency of the signal can be determined as follows,

Height of the display is 4.6cm

therefore, peak - to - peak voltage = 4.6cm 0:5V/cm = 2.3V

Width of a complete signal is 7cm

therefore, period of the signal = 7cm 100 10^{6}s/cm = 700 x 10^{6}s

Therefore, frequency of signal = $\frac{1}{700 \quad 10^{6}s}$ = 1430Hz

Measurement of phase: Consider two input signals of the same frequency one connected to the X-plates and the other to the Y-plates. An ellipse will be seen on the screen as shown on Figure 3.15. The trace is centred and the peak vertical displacements of the ellipse y_1 and y_2 are measured.

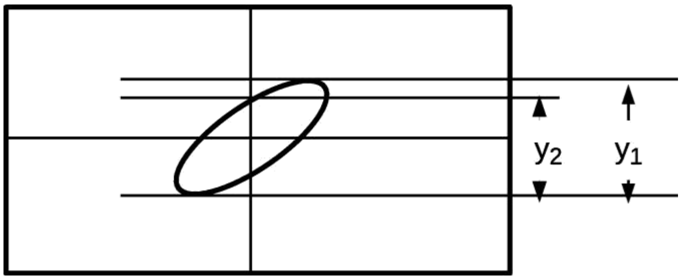


Figure 3.15: Measurement of phase on a CRO.

Suppose the displacements in x and y are given by

$$x = a \sin !t \qquad y = y_1 \sin(!t + \ )$$

where a and y_1 are the amplitudes in the x and y directions respectively, is the phase angle. When x =0,

$$\sin ! t = 0 \qquad \text{and} \qquad y = y_2 = y_1 \sin$$

hence,

$$\sin \qquad = \frac{y_2}{}$$

from which can be determined.

The patterns formed when two sine waves or sinuisoidal signals are applied simultaneously to the vertical and horizontal de ecting plates of the CRO (Y- and X- plate) are known as Lissajous[1] gures. The shape of the pattern depends on the frequency and phase relationship of the two sine waves, in addition to their respective amplitudes. Lissajous gures are used for;

determine an unknown frequency by comparing it with a known frequency,

check audio oscillators with a known frequency signal, and,

check audio ampli ers and feedback networks for phase shift.

[1]named in honour of the french scientist who rst obtained them geometrically

When an unknown waveform is applied to the Y-input and a known waveform to the X-input, the frequency of the known waveform is adjusted until an exact relationship is achieved between the two frequencies as illustrated on Figure 3.16. If a vertical and a horizontal line are imagined as being drawn at the side and top of the trace then

$$\frac{f_x}{f_y} = \frac{\text{no: of loops touching verical line}}{\text{no of loops touching horizontal line}}$$

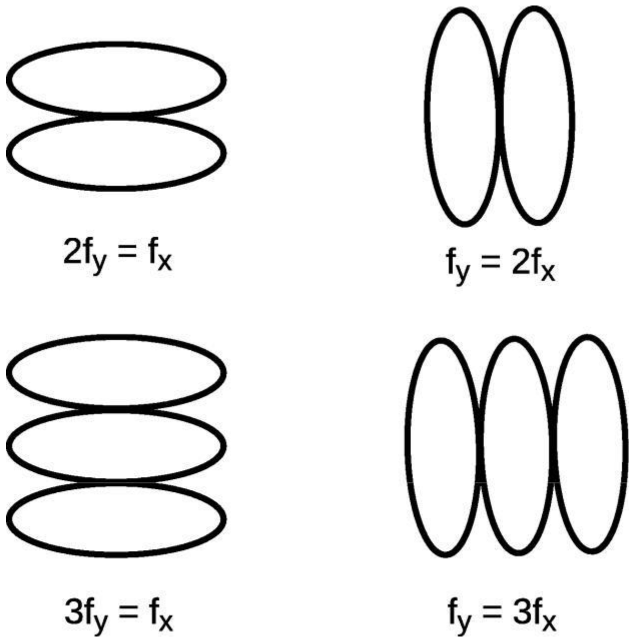


Figure 3.16: The use of Lissajous gures to measure the frequency of a signal on a CRO.

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

SPH101 Electricity and Magnetism I

Lecture No. 9.
Outline

- resistivity and conductivity
- ohmic and non-ohmic conductors

Resistivity and Conductivity: Di erent materials o er di erent levels or magnitudes of resistance to the ow of electrical charges or electrical current i.e they have di erent resistance values. If a potential di erence V is ap-plied between 2 points of the conductor and a current i ows across it, the resistance R of the material is

$$R = \frac{V}{i} \qquad (3.22)$$

$$1 \text{ Ohm} = \frac{1 \text{ volt}}{\text{ampere}}$$

The resistivity of the material of the conductor is de ned as

$$= \frac{E}{j} \quad :m \qquad (3.23)$$

where

- E (V/m) is the electric eld strength applied.
- $j(A/m^2)$ is the electric current density.

In vector form

$$\tilde{E} = \tilde{j} \qquad (3.24)$$

Equations 3.23 and 3.24 are valid for isotropic materials, whose electrical properties are same in all directions. Resistivities of some materials at room temperature (20^oc) is shown on Table 3.1

Table 3.1: Resistivity of some materials at room temperature (20^{o}c)

Material	Resistivity (:m)	temperature coe cient of resistivity (per^{o}c)
typical metals		
Silver	1.62 x 10^{8}	4.1 x 10^{3}
Copper	1.69 x 10^{8}	4.3 x 10^{3}
Aluminium	2.75 x 10^{8}	4.4 x 10^{3}
Tungsten	5.25 x 10^{8}	4.5 x 10^{3}
Iron	9.68 x 10^{8}	6.5 x 10^{3}
Platinum	10.6 x 10^{8}	3.9 x 10^{3}
Manganin	48.2 x 10^{8}	0.002 x 10^{3}
typical semiconductors		
Silicon(pure)	2.5 x 10^{3}	-70 x 10^{3}
Silicon n-type	8.7 x 10^{4}	
Silicon p-type	2.8 x 10^{3}	
typical Insulators		
Glass	10^{10} - 10^{14}	
Polystyrene	>10^{14}	
Fused quartz	’ 10^{6}	

The conductivity of a material is the reciprocal of it’s resistivity, i.e

$$= \frac{1}{} \qquad (:m)^{1}$$

so that Equation 3.24 can be written as

$$\tilde{j} = \tilde{E} \qquad (3.25)$$

Consider a cylindrical conductor, of uniform cross-sectional area A and length L carrying a steady current i with a potential di erence V between it’s ends. If the cylinder cross-sections at each end are equipotential surfaces, the elec-tric eld and the current density are constants for all points in the cylinder and have the values,

$$E = \frac{V}{L} \qquad \text{and} \qquad j = \frac{i}{A}$$

The resistivity is

$$= \frac{E}{j} = \frac{\frac{V}{L}}{\frac{i}{A}} \qquad \text{but} \quad \frac{V}{i} = R$$

hence

$$R = \frac{L}{A} \qquad (3.26)$$

Equation 3.26 applies only to a homogeneous, isotropic conductor of uniform cross-section subject to a uniform electric eld.

Example 1: A rectangular block of iron has dimensions 1.2cm 1.2cm 15cm. The resistivity of the iron at room temperature is 9.68 x 10^{8} :m:

The resistance of the block measured between the two square ends is,

$$R = \frac{L}{A} = 9{:}68\ 10^{8}\ {:}m\ \frac{0{:}15m}{(1{:}2\ \ 10^{2}m)^{2}}$$

$$= 100{:}83$$

The resistance between the two opposing square faces,

$$R = \frac{L}{A} = 9{:}68\ \ 10^{8}\ {:}m\ \frac{1{:}2\ \ 10^{2}m}{(1{:}2\,10^{2}m\ \ 15\ \ 10^{2}m)}$$

$$= 0{:}65$$

Ohms Law: George Simon Ohm (1781 - 1854) investigated how the current I in a given metal varied with the p.d V across it. An experimental set-up for this investigation is illustrated on Figure 3.17.

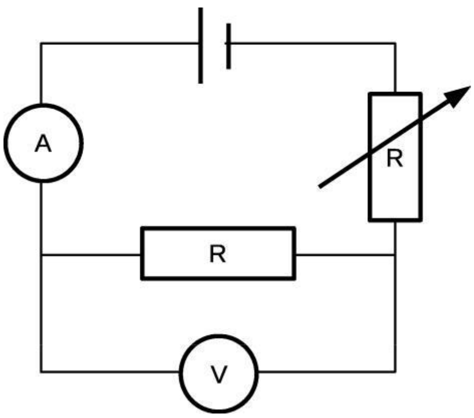


Figure 3.17: Circuit diagram for the investigation of the current and voltage in conductors

The variable resistor or rheostat provides a varying current through the cir-cuit and the potential di erence across the resistor R is measured at these di erent currents. Ohm observed that the potential di erence across a resis-tor R is proportional to the current through it; i.e

$$V / I$$

known as Ohm's Law, which states,

under constant physical conditions, the resistance V/I is a constant indepen-dent of V or I

and holds for many conductors.

Ohmic and non-Ohmic conductors: Ohm's Law is obeyed by metals, also called Ohmic conductors. In this type of conductors the direction of current is reversed when the p.d V is reversed. The characteristic or I-V graph is thus a straight line passing through the origin Non-Ohmic conduc-tors are those which do not obey Ohm's Law (V / I). Their I-V graphs may have a curve instead of a straight line, or it may not pass through the origin as in the ohmic characteristic; or it may conduct poorly or not at all when the p.d is reversed (-V)

Example 2: Calculate =A for a 14-gauge copper wire and hence determine the electric eld strength in a 14-gauge copper wire carrying a current of 1A. The diameter of a 14-gauge copper wire is 0.163cm and = 1.7 x 10 ^{8} :m

$$A = \frac{d^2}{4} = \frac{}{4}(0{:}163 \;\; 10^{\;2}m)^2 = 2{:}09 \;\; 10^{\;6}m^2$$

thus

$$\frac{}{A} = \frac{1{:}7 \;\; 10^{8} \;\; {:}m}{2{:}09 \;\; 10^{6}m^2} = 8{:}146 \;\; 10^{\;3} \;\; {=}m$$

Hence for a 1m long 14-gauge wire R = 8.146 x 10 ^{3} and the voltage drop when carrying 1A is

$$V = IR = 1A \; 8{:}146 \; 10^{\;3} = 8{:}15 \; 10^{\;3}V$$ and the

electric eld strength

$$E = \frac{V}{L} = \frac{8{:}15 \;\; 10^{\;3}V}{1m} = 8{:}15 \;\; 10^{\;3}\frac{V}{m}$$

Example 3: A wire of length 1m has a resistance of 0.3 . It is uniformly stretched to a length of 2m. What is it's new resistance?

The resistivity of the material does not change when the wire is stretched.

$$= \frac{R_1 : A_1}{l_1} = \frac{R_2 : A_2}{l_2}$$

Given $R_1 = 0.3$, $l_1 = 1m$, $l_2 = 2m = 2l_1$. Let the diameter of the wire be d so that

$$A_1 = \frac{}{4} d_1^2 \qquad A_2 = \frac{}{4} d_2^2$$

Since the volume of the wire does not change

$$\frac{}{4} d_2^2 l_2 = \frac{}{4} d_1^2 l_1$$

$$d_2^2 = d_1^2 \frac{l_1}{l_2} = d_1^2 \frac{l_1}{2 l_1} = \frac{d_1^2}{2}$$

Then

$$\frac{R_2 A_2}{l_2} = \frac{R_1 A_1}{l_1}$$

$$\frac{R_2 \frac{}{4} d_2^2}{2 l_1} = \frac{R_1 \frac{}{4} d_1^2}{l_1}$$

hence

$$\frac{R_2 \frac{d_1^2}{2}}{2} = \frac{R_1 d^2}{1}$$

$$R_2 = 4R_1 = 4 \quad 0:3 = 1:2$$

Example 4: A 16-gauge copper wire (diameter = 1.29mm) can safely carry a maximum current 6A (assuming rubber insulation)

a) What is the maximum potential di erence that can be safely applied accross 40m of such a wire.

b) Find the current density and electric eld in the wire which it carries 6A.

c) Evaluate the power dissipated in the wire when it carries 6A.

$_{copper}$ = 1.7 x 10 ^{8} m

$$R = \frac{l}{A} \qquad = 1{:}7\ 10^{8} \frac{40m}{\frac{}{4}(1{:}2\quad 10\ ^{3}m)^{2}}$$

$$= 0{:}52$$

$$V_{max} = i_{max}\ R = 6A\ 0{:}52 \ = 3{:}12V$$

The current density

$$j = \frac{I}{A} = \frac{6A}{\frac{}{4}(1{:}29\quad 10\ ^{3}m)^{2}} = 4{:}59\ \ 10^{6}A{=}m^{2}$$

The electric eld

$$E = \frac{V}{l} = \frac{3{:}12V}{40m} = 0{:}078\ V{=}m$$

The power dissipated

$$p = V\ I = 3{:}12V \qquad 6A = 18{:}72\ W$$

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

Chapter 4

The Magnetic Field

SPH101 Electricity and Magnetism I

Lecture No. 10.
Outline

- the magnetic eld

The magnetic eld: The space around a permanent magnet or a current-carrying conductor is described as the location of a magnetic eld $\tilde{H}$. The magnetic ux density is denoted by $\tilde{B}$ (although in many cases it is also refered to as magnetic eld). Figure 4.1 shows iron llings sprinkled on a sheet of paper covering a bar magnet. The distribution of the llings indicates the pattern of lines of the magnetic eld. Similarly, a radial pattern of magnetic eld lines will be set up by a current-carrying conductor. The moving electric charge or an electric current sets up a magnetic eld, which can then exert a magnetic force on other moving charges or currents.

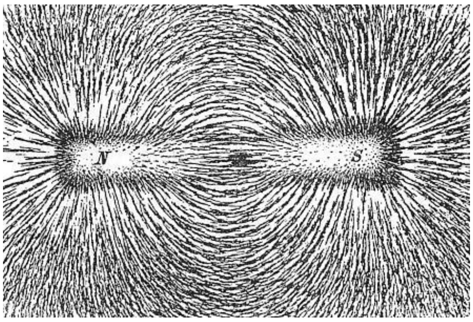


Figure 4.1: Lines of the magnetic eld displayed using iron lings.

The magnetic force on a moving charge: Consider the projection of a test charge q through a point p with a velocity $\tilde{V}$ as illustrated on Fiure 4.2. The direction of projection is at an angle with the magnetic eld $\tilde{B}$. The test charge experiences a magnetic force $\tilde{F}$ which is perpendicular to the plane of $\tilde{B}$ and $\tilde{V}$, whose magnitude is

$$F = q V B \sin \qquad (4.1)$$

which can be written in vector form as

$$\tilde{F} = q \tilde{V} \quad \tilde{B} \qquad (4.2)$$

so that $F_{max} = qvB$ when $= 90^0$ and $F_{min} = 0$ when $= 0$ or 180^0.

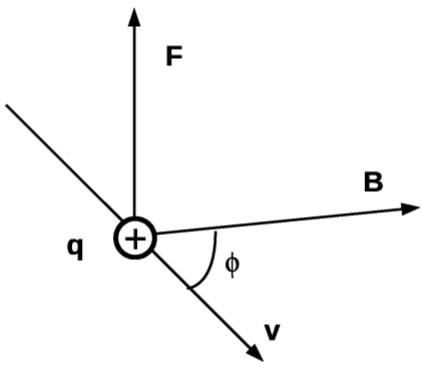


Figure 4.2: Motion of a test charge magnetic eld.

The SI unit for the magnetic eld is the tesla (T)

$$1 \text{ tesla} = \frac{1 \text{ newton}}{\text{coulomb meter=second}} = \frac{1 \text{ newton}}{\text{ampere meter}}$$

An earlier (non SI) unit for was the gauss

$$1 \text{ tesla} = 10^{4} \text{gauss} :$$

Table 4.1: Typical values of some magnetic elds

Location	Magnetic eld $\tilde{B}$(T)
At the surface of a neutron star (calculated)	10^{8}
Near a super conducting magnet	5
Near a large electromagnet	1
Near a small bar magnet	10^{2}
At the surface of the earth	10^{4}
In the interstellar space	10^{10}
In a magnetically shielded room	10^{14}

Magnetic eld lines: Magnetic eld lines are lines of magnetic force originating from one end (North pole) of a magnet and ending at the opposite end (South pole) of the magnet as illustrated on Figure 4.3

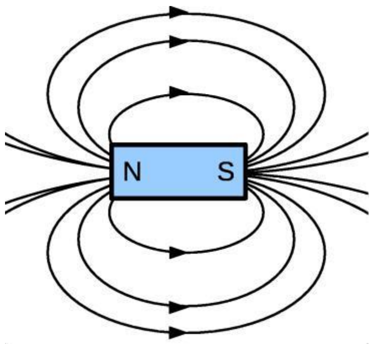


Figure 4.3: The magnetic eld lines for a bar magnet.

The magnetic eld lines form closed loops leaving the magnet at it's north pole and entering it at it's south pole. Studies on the properties of magnets conducted several centuries ago showed that like poles repel and unlike pole attract. It is interesting to note that magnet that is freely suspended would always align itself in the direction of the external eld. A magnetic compass always point to the geographic north. Thus there exists a magnetic eld due to the earth with the Antarctica and Arctic regions being the north and south poles respectively as illustrated on Figure 4.4

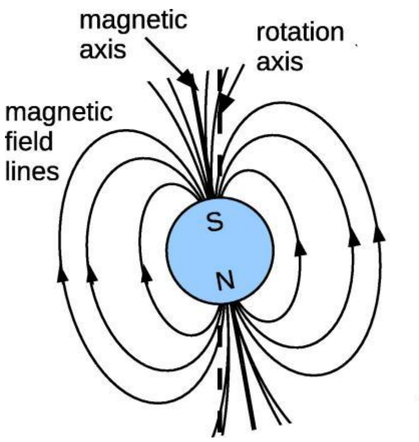


Figure 4.4: The Earth's magnetic eld. Note that the geographic north corresponds to the magnetic south pole.

The following are di erent types of magnetic materials.

Paramagnetic - have permanent magnetic dipole moments which do not interact strongly with each other and are normally randomly ori-ented. In the presence of an external magnetic eld, the dipoles are partially aligned in the direction of the external eld, thereby increas-ing the eld.

Ferromagnetic - have strong interaction between neighbouring mag-netic dipoles and a high degree of alignment is achieved with weak external magnetic elds thereby causing a very large increase in the total eld. Even when there is no external magnetic eld, ferromag-netic materials may have magnetic dipoles aligned like in permanent magnets.

Diamagnetic - result from induced magnetic moment opposite in di-rection to the external eld. The induced dipoles thus weaken the resultant magnetic elds.

A soft iron core is an example of a ferromagnetic material and can therefore be magnetised when aligned to the earth's magnetic eld.

Example 1: A uniform magnetic eld $\tilde{B}$ with magnitude 1.2mT, points ver-tically upwards throughout the volume of the room in which you are sitting. A 5.3MeV proton moves horizontally from south to north through a certain point in the room. What magnetic de ecting force acts on the proton as it passes through this point? The proton mass is 1.6 x 10^{27}kg.

The speed of the proton is calculated from $k = \frac{1}{2} mV^2$

$$V = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\,(5.3\ \text{Mev})(1.6\ 10^{13}\ \text{J/MeV})}{1.67\ 10^{27}\ \text{kg}}}$$

$$= 3.2\ 10^{7}\ \text{m/s}$$

The force on the proton is calculated as follows,

$$F = qVB\sin\theta$$

$$= (1.6 \times 10^{-19} C)(3.2 \times 10^{7} m/s)(1.2 \times 10^{-3} T)\sin 90^{0}$$

$$= 6.1 \times 10^{-15} N$$

so that the acceleration is,

$$a = \frac{F}{m} = \frac{6.1 \times 10^{-15} N}{1.67 \times 10^{-27} kg} = 3.7 \times 10^{12} m/s^{2}$$

Lorentz Force: Consider the motion of a charged particle passing through a region in which the $\vec{E}$ and $\vec{B}$ fields are perpendicular to each other and to the velocity of the particle as illustrated on Figure 4.5.

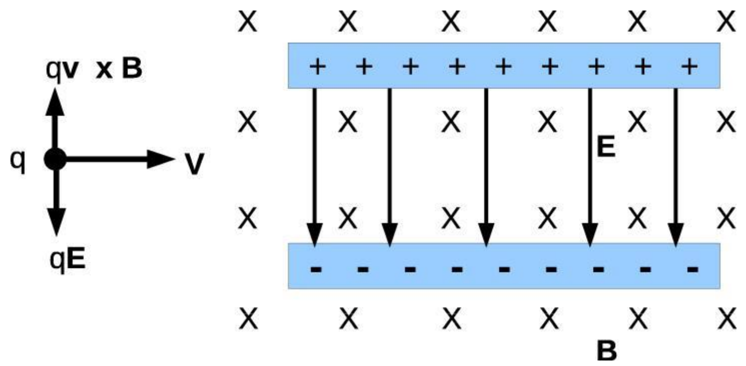


Figure 4.5: The motion of a charged particle in uniform electric magnetic fields. The magnetic field is indicated to be through the paper.

In the presence of both electric field $\vec{E}$ and a magnetic field $\vec{B}$, the total force on the charged particle can be expressed as

$$\vec{F} = q\vec{E} + q\vec{V} \times \vec{B} \qquad (4.3)$$

known as Lorentz force.

The electric and magnetic elds can be adjusted until the magnitudes of the forces are equal, in which case the Lorentz force is zero. In scalar terms

$$qE = q\,V\,B \text{or} \quad V = \frac{E}{B}$$

Only particles with this speed will pass through the region unde ected, others will be de ected. This value is independent of the change or mass or the particles. The crossed E and B elds can therefore serve as a velocity selec-tor; only particles with speed V = E/B pass through the region unde ected by the two elds, while particles with other velocities are de ected.

Assignment: Show how Sir J.J. Thompson applied this technique to deter-mine the charge-to-mass ratio of the electron.

Motion of a particle in a magnetic eld: In the special case when the velocity of the particle is perpendicular to a uniform eld the particle moves in a circular circuit,

$$q\,V\,B = \frac{mV^2}{r} \quad \text{or} \quad r = \frac{mV}{qB}$$

The angular frequency of the circular motion is

$$! = \frac{V}{r} = \frac{qB}{m} \qquad (4.4)$$

known as the cyclotron frequency. The periodic time is ,

$$T = \frac{2}{!} = \frac{2\ m}{q\,B} \qquad (4.5)$$

Some of the many interesting applications of the circular motion of charged particles in a uniform magnetic eld are

Mass spectrograph - identi cation of radoisotopes

Cyclotron - acceleration of particles to cause nuclear reaction for basic research and isotope production for medical/industrial applications

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

SPH101 Electricity and Magnetism I

Lecture No. 11.
Outline

- Ampere's Law
- Self inductance

Amperes Law: As already observed, the motion of charged particles or the ow of electric current creates a magnetic eld as illustrated on Figure 4.6.

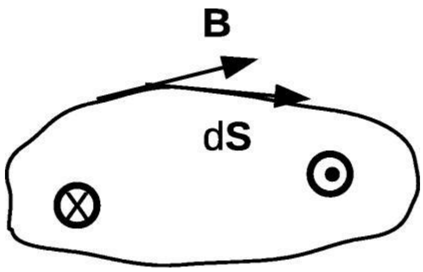


Figure 4.6: A closed loop for the calculation of the magnetic eld

The magnetic eld due to a current i can be calculated as follows,

$$\oint \tilde{B} : d\tilde{S} = \mu_o i \qquad (4.6)$$

which is a line integral of a closed path or Amperian loop of the current. For any closed loop, the integrated magnetic eld around the loop is propor-tional to the electric current in the loop creating the magnetic eld. This is known as Ampere's law[1]. The constant of proportionality μ_o is known as the permeability constant.

Example 1: Consider a long straight wire carrying a current i through a plane of paper as illustrated on Figure 4.7.

[1] discovered by Andre-Marie Ampere in 1826

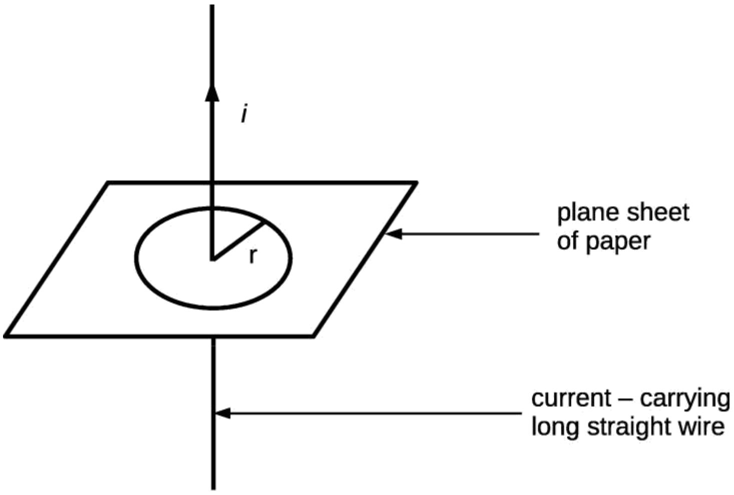


Figure 4.7: The magnetic field due to a current-carrying conductor

A circular Amperian loop is used to define the field set up by the $\tilde{B}$ has only tangential component hence $\theta = 0$

$$\oint B\, dS \cos\theta = B \oint dS = B\, 2\pi r$$

$$B\, 2\pi r = \mu_o i$$

$$B = \frac{\mu_o i}{2\pi r}$$

Energy Density and the magnetic field: The energy density in a uniform magnetic field is given by

$$u_B = \frac{1}{2\mu_o} B^2 \qquad (4.7)$$

which can be derived by considering the magnetic field $\tilde{B}$ in a solenoid of cross-section A and length l and inductance L.

$$u_B = \frac{U_B}{A\, l} \qquad \text{but } U_B = \frac{1}{2} L\, i^2$$

hence

$$u_B = \frac{\frac{1}{2} L\, i^2}{A\, l}$$

but

$$L = \ _o n^2 \, l \, A$$

$$B = \ _o \, i \, n$$

$$i = \frac{B}{_o \, n}$$

so,

$$_B = \frac{\frac{1}{2} (_o n^2 \, l \, A) B^2}{A \, l (_o^2 n^2)}$$

$$_B = \frac{1}{2 \ _o} B^2 \qquad (4.8)$$

Example 2:Compare the energy required to set-up in a cube of 10cm on edge

a) a uniform $\tilde{E} = 1.0 \text{ x } 10^5$ V/m and

b) a uniform $\tilde{B} = 1.0$T.

the permeability costant is given as

$$_o = 4 \quad 10^{7} T{:}m{=}A = 1{:}26 \quad 10^{6} H{=}m$$

$$U_E = \ _E V_o = \frac{1}{2} \ E^2 \, V_o$$

$$= (0{:}5)(8{:}9 \ 10^{12} c^2{=}N{:}m^2)(10^5 V{=}m \ 0{:}1m^3)$$

$$4{:}5 \ 10^{5} J$$

$$U_B = \ _B V_o$$

$$= \frac{B^2}{2 \ _o} V_o = \frac{(1{:}0T)^2 (0{:}1m)^3}{2(4 \quad 10^{7} T{:}m{=}A)} = 400J$$

$$) \frac{_B}{_E} \quad 10^7$$

more energy can be strored in a magnetic eld.

Solenoids: A solenoid is a long wire wound in a close-packed helix carrying a current i as illustrated on Figure 4.8. The helix is very long compared to its diameter so as to have uniform magnetic eld lines.

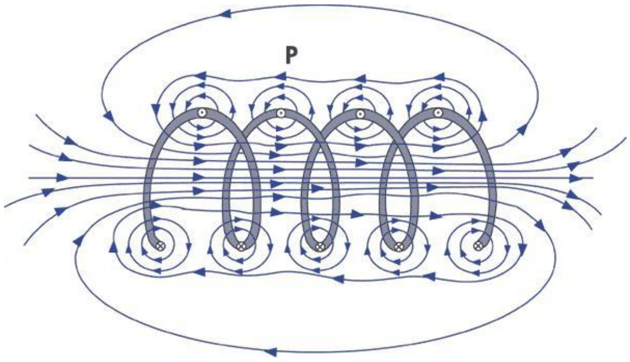


Figure 4.8: The magnetic eld lines in a solenoid.

The solenoid eld is a vector sum of the elds set-up by all the turns that make up the solenoid. In the limiting case of tightly packed square wires, the solenoid becomes essentially a current sheet and the eld B at outside points approaches zero as the solenoid approaches an in nitely long cylindri-cal current sheet as illustrated on Figure 4.9.

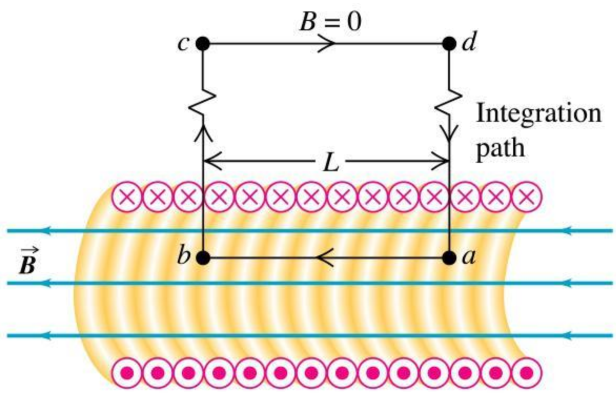


Figure 4.9: Calculation of the magnetic eld in a solenoid

An Amperian loop (rectangle abcd) can be used to calculate the magnetic field of this long idealised solenoid

$$\oint \vec{B}\cdot d\vec{S} = \int_a^b \vec{B}\cdot d\vec{S} + \int_b^c \vec{B}\cdot d\vec{S} + \int_c^d \vec{B}\cdot d\vec{S} + \int_d^a \vec{B}\cdot d\vec{S}$$

$$= B\,h(+0+0+0)$$

for n turns in the section of length h

$$i = i_0\,n\,h \qquad i_0 = \text{current through solenoid}$$

hence

$$B = \mu_0\, i_0\, n \tag{4.9}$$

is the magnetic field inside a solenoid.

Assignment: Show that for a toroid,

$$B = \frac{\mu_0 i_0 N}{2\pi r}$$

Example 3: A solenoid has a length L = 1.23m and an inner diameter d = 3.55cm. It has five layers of windings of 850 turns each and carries a current i_0 = 5.57A. Calculate the magnitude of the magnetic field $\vec{B}$ at it's centre

$$B = \mu_0\, i_0\, n = (4\pi \times 10^{-7}\,T{\cdot}m/A)(5.57A)\left(\frac{5\ \ 1.23m}{859\ \text{turns}}\right)$$

$$= 2.42 \times 10^{-2}T = 24.2mT$$

Example 4: A solenoid 1.33m long and 2.60cm in diameter carries a current of 17.8A. The magnetic field inside the solenoid is 2.4mT. Find the length of wire forming the solenoid.

$$B = \mu_0 i_0 n \quad \Rightarrow N = \frac{BL}{\mu_0 i} \text{ and } n = \frac{N}{L}$$

the total number of turns

$$N = \frac{B\ L}{_o\ i} = \frac{(22{:}4\ \ 10^{3}T)(1{:}33m)}{(4\ \ 10^{7}T{:}m{=}A)(17{:}8A)}$$

$$= 1331{:}89 \text{ turns}$$

The length of the wire

$$L_W = N(\ D) = 1331{:}89 \qquad \frac{2{:}60cm}{100}$$

$$' \ 109m \text{ long wire}$$

Faraday's law of induction: The magnetic ux is a measure of the number of magnetic eld lines passing through any surface.

$$_o = \int B{:}d\tilde{A}$$

$d\tilde{A}$ = an element of area of the surface

The SI unit of the magnetic ux is the tesla.m^2 i.e

1Weber = 1 tesla.m^2

The following two experimental observations demonstrate the induction of an e.m.f in a circuit.

1. A moving magnet: An ammeter de ects when a magnet is moved through a coil, showing that a current has been set-up in the coil. Further experiments show that it is the relative direction of the magnet and coil that sets-up the induced current which is said to be due to an induced electromotive force.
2. A changing current: A change in current through a coil causes an ammeter to de ect momentarily.

In the above examples, it is the moving magnet or changing current that is responsible for the induced e.m.fs. The mathematical basis of induced e.m.f was studied by Michael Faraday in 1831 and formulated the law of induction, The induced e.m.f in a circuit is equal to the negative of the rate at which the magnetic ux through the circuit is changing with time

$$" = \frac{d}{dt}{}_{B} \quad (4.10)$$

where " = induced e.m.f for a coil of N turns tightly wound to experience some change of ux,

$$" = N \frac{d}{dt}{}_{B} \quad (4.11)$$

Self-inductance of a solenoid: The magnetic eld inside the solenoid is uniform and given by

$$B = {}_{o} n I = {}_{o} \frac{N}{l} I \qquad (4.12)$$

where

$n = \frac{N}{l}$ - is no. of turns per unit length

N = total no. of turns

l = total length

I = current in solenoid

The total magnetic ux is

$${}_{m} = B\,N\,A = {}_{o} n\,I\,N\,A$$

so that with N = n l

$${}_{m} = {}_{o} n^{2} (l\,A)\,I$$

showing that,

$${}_{m} / I$$

and,

$$L = {}_{o} n^{2} A\,l \qquad (4.13)$$

is the constant of proportionality known as the self- inductance of the coil.

The inductance of a circuit element (such as a solenoid) can therefore be de ned, using Faraday's law,

$$"_l = L \frac{dI}{dt} \qquad (4.14)$$

where $"_l$ is the e.m.f. across the inductor. The SI unit for inductance is the henry[2] (H)

$$1H = 1 \text{ volt} \quad \text{second} = \text{ampere}$$

Example 5: Find the self-inductance of a solenoid of length 10cm, area 5cm ^{2} and 100 turns.

$$n = \frac{N}{l} = \frac{100 \text{ turns}}{0{:}1m} = 10^3 \text{ turns=m}$$

hence

$$L = (4 \ 10^{\ 7})(10^3 \text{ turns})^2 (5 \ 10^{\ 4} m^2)(0{:}1m) = 6{:}28 \quad 10^{\ 5} H$$

Example 6: At what rate must the current in the solenoid above change to induce an e.m.f of 10V?

from Faraday's Law

$$" = \frac{d}{dt} \ _m \quad \text{but} \quad _m = L I \qquad \text{so that} \quad " = L \frac{dI}{dt} = 10V$$

hence

$$\frac{dI}{dt} = \frac{"}{L} = \frac{10V}{6{:}28 \quad 10^{\ 5}H} = 1{:}6 \ 10^5 \text{ A=s}$$

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

Dr. N.O. Hashim

[2] after Joseph Henry (1797 - 1878)

SPH101 Electricity and Magnetism I

Lecture No. 12.
Outline

series and parallel arrangement of inductors, L-R circuits

energy storage in magnetic elds, the coaxial cable

Series and parallel arrangement of inductors: Figure 4.10 shows series and parallel arrangement of inductors.

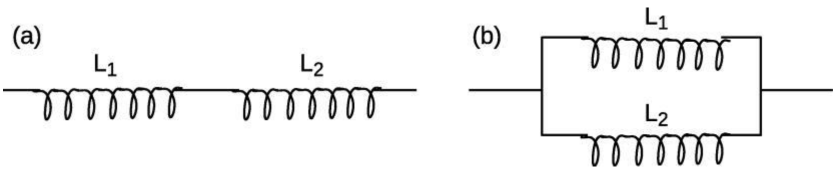


Figure 4.10: Arrangement of inductors in (a) series and (b) parallel.

The total inductance is calculated as follows, for the series arrangement

$$L=L_1+L_2 \qquad (4.15)$$

and for the parallel arrangement

$$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}: \qquad (4.16)$$

L-R Circuits: Consider an inductor L connected in series to a resistor R and source of e.m.f " as illustrated on Figure 4.11.

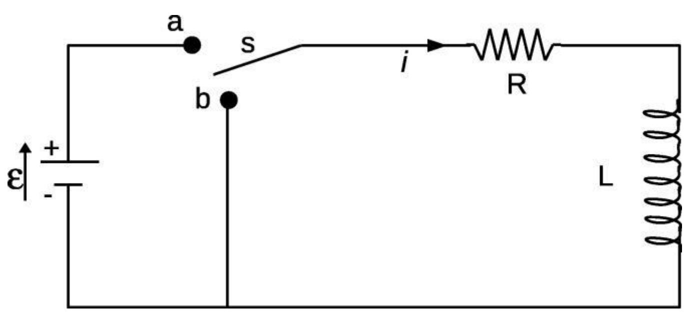


Figure 4.11: A simple L-R circuit.

When the switch is thrown to a, the loop theorem gives,

$$i R \quad L \frac{d i}{dt} \quad + " = 0 \tag{4.17}$$

$$L \frac{d i}{dt} + i R = "$$

whose solution is of the form

$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \qquad \text{with } \tau_L = \frac{L}{R} \qquad (4.18)$$

with the properties

$i = 0$ at $t = 0$

$i = \frac{\varepsilon}{R}$ as $t \to \infty$

The quantity $\tau_L = \frac{L}{R}$ represents the inductive time constant, and determines how rapidly the current approaches the steady value $\frac{\varepsilon}{R}$. The variation of potential difference across the resistor R and inductor L is illustrated on Figure 4.12

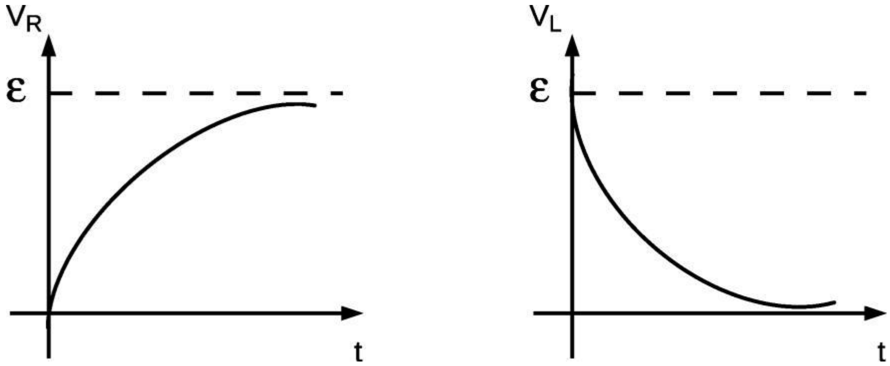


Figure 4.12: The variation of voltage across the resistor and inductor in a simple L-R circuit.

Note that:

at t = 0 V_R = 0, V_L = "

as t ! 1 V_R ! " V_L ! 0

at any time V_R + V_L = "

If the switch is thrown to b, the loop theorem gives

$$L \frac{di}{dt} + i R = 0 \qquad (4.19)$$

whose solution is

$$i(t) = i_o e^{t=}{}_{L} \qquad (4.20)$$

where i_o is the current at t=0. The decrease in current occurs at the same exponential time constant $_L = \frac{L}{R}$, as does the rise in the current.

Example 1: A solenoid has an inductance of 53mH and resistance 0.37 . If it is connected to a battery, how long will it take for the current to reach one-half it's full equilibrium value?

Equilibrium value of current i = $\frac{"}{R}$ reached as t ! 1

$$\frac{1}{2} \frac{"}{R} = \frac{"}{R} \quad 1 \; e^{t= L}$$

hence,

$$t = {}_{L} \ln 2 = \frac{L}{R} \ln 2 = \frac{53 \;\; 10^{3} H}{0:37} \quad \ln 2 = 0:1s$$

Energy storage in a magnetic eld: Consider a series L-R circuit illustrated on Figure 4.13.

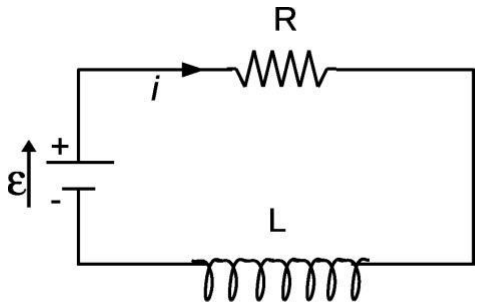


Figure 4.13: A simple L-R circuit.

From the loop theorem

$$\varepsilon = i R + L \frac{di}{dt}$$

hence

$$\varepsilon i = i^2 R + L i \frac{di}{dt} \qquad (4.21)$$

where,

εi = rate at which seat of e.m.f delivers energy to circuit.

$i^2 R$ = rate at which energy is dissipated in resistor

$L i \frac{di}{dt}$ = rate at which energy is stored in the magnetic eld.

Let U_B represent energy stored in the magnetic eld of the inductor L,

$$\text{then } \frac{dU_B}{dt} = L i \frac{di}{dt}$$

$$U_B = \int_0^{U_B} dU_B = \int_0^{i} L i \, di = \frac{1}{2} L i^2$$

$$U_B = \frac{1}{2} L\, i^2 \qquad (4.22)$$

is the total stored magnetic in an inductance L carrying a current i. For a uniform magnetic eld, the stored energy is uniformly distributed and the energy density is,

$${}_B = \frac{U_B}{A\,l} \qquad \text{where} \qquad A\,l = \text{volume of inductor}$$

and since

$$U_B = \frac{1}{2} L\, i^2$$

then,

$${}_B = \frac{\frac{1}{2} L\, i^2}{A\,l} : \qquad (4.23)$$

For a solenoid,

$$L = {}_o\, n^2\, l\, A$$

so that

$${}_B = \frac{1\ {}_o\, n^2\, l\, A\, i^2}{2A\,l} = \frac{1}{2\ {}_o}\ {}_o^2\, n^2\, i^2$$

and with,

$$B = {}_o\, n\, i$$

$${}_B = \frac{1}{2\ {}_o} B^2 \qquad (4.24)$$

which is true for all magnetic con gurations even through the derivation was based on a solenoid.

Example 2: A coil has an inductance of 53 mH and resistance 0.35 Ω.

(a) If a 12 V source of e.m.f. is applied across the inductor, how much energy is stored in the magnetic field after the current has reached it's maximum value?

(b) In terms of the inductive time constant τ_L, how long does it take for the stored energy to reach half of it's maximum value?

The maximum current is calculated as follows,

$$i_{max} = \frac{\varepsilon}{R} = \frac{12V}{0.35\,\Omega} = 34.3\ A$$

hence the stored energy at $i = i_{max}$ is,

$$U_B = \frac{1}{2}Li^2_{max} = \frac{1}{2}(53 \times 10^{-3}H)(34.3A)^2 = 31\ J$$

Let i be the current at the instant when the stored energy has reached half it's maximum value, then,

$$\frac{1}{2}Li^2 = \frac{1}{2}\,\frac{1}{2}Li^2_{max}$$

hence,

$$i = \frac{i_{max}}{\sqrt{2}} = \frac{\varepsilon}{R\sqrt{2}}$$

so that from the expression for the current,

$$i = \frac{\varepsilon}{R}\left(1 - \exp(-t/\tau_L)\right) = \frac{\varepsilon}{R}\frac{1}{\sqrt{2}}$$

one gets,

$$t \simeq 1.23\,\tau_L.$$

The coaxial cable: consists of two long concentric conductors as illustrated on Figure 4.14. The conductors have radii a and b where b>>a. It's central conductor carries a steady current i, and the outer conductor provides the return path.

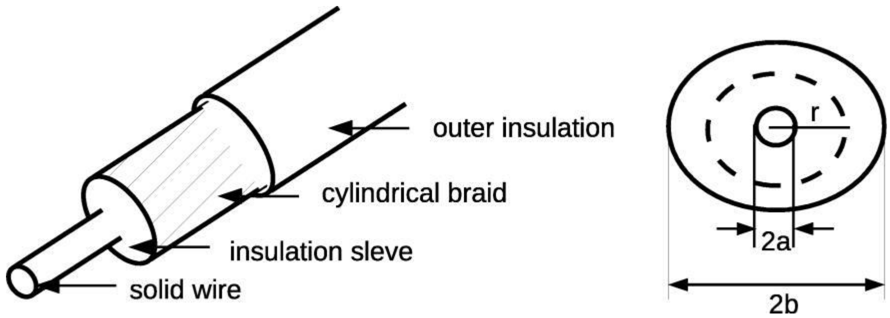


Figure 4.14: The cross-section of a coaxial cable.

In the sphere between the two conductors, Amperes law can be used to evaluate the magnetic eld in the cable,

$$\oint \vec{B}\cdot d\vec{S} = \mu_0 i$$

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} \qquad (4.25)$$

The energy density η_B for points between the conductors

$$\eta_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2$$

hence,

$$\eta_B = \frac{\mu_0 i^2}{8\pi^2 r^2} \qquad (4.26)$$

Consider a volume element dV consisting of a cylidrical shell whose radii is r and r + dr and whose length is l. The energy dU_B contained in it is

$$dU_B = U_B\, dV = \frac{\mu_o i^2}{8\pi^2 r^2}(2\pi r l)\, dr$$

$$= \frac{\mu_o i^2 l}{4\pi}\frac{dr}{r}$$

and the total energy stored

$$U_B = \int dU_B = \frac{\mu_o i^2 l}{4\pi}\int_a^b \frac{dr}{r}$$

$$U_B = \frac{\mu_o i^2 l}{4\pi} \ln\frac{b}{a} \qquad (4.27)$$

Using the earlier result,

$$U_B = \frac{1}{2} L i^2$$

the inductance of the cable is given by,

$$L = \frac{2U_B}{i^2} = \frac{\mu_o l}{2\pi} \ln\frac{b}{a}: \qquad (4.28)$$

The coaxial cable has useful applications in electrical/electronic signal trans-mission. Besides the inductance of the cable one has also to consider its impedance. Any impedance mismatches in a circuit may lead to undesired signal re ections or even losses.

NB: These notes are an outline of what is discussed during the Lecture. Students are encouraged to actively attend lectures and most importantly, solve as many examples as possible on their own.

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