SCHOOL OF PURE AND APPLIED SCIENCES

COMPLEX ANALYSIS

DATE: 25/8/2017 **TIME: 11:00 – 1:00 pm**

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE (30MARKS)

a. State the Cauchy Riemann equations in polar form (4marks)

b. Find the value of

$$\left(1-\sqrt{3}i\right)^{90}$$ (3marks)

i) $$\left(-1-\sqrt{3}i\right)^{\frac{1}{2}}$$ (3marks)

c. Evaluate the following limit $$\lim_{x+iy\to 0}\frac{x+y-1}{z}$$ (4marks)

d. Evaluate

i) The conjugate of $z_1 z_2$

ii) $z_1 + z_2$ where $z_1 = x+iy$, $z_2 = 2+i3$ (4 marks)

e. Express the following equation in terms of conjugate coordinates $2x+3y=5$

(4 marks)

f. Express the complex number $-3+3i$ in polar form and draw the vector associated to this number in complex plane. (4marks)

g. Determine the continuity of the function $\frac{z^2+9}{z(z-3i)}$ at $z=0$ and $z=3\mathbf{i}$ (4marks)

QUESTION TWO (20MARKS)

a. Test for analyticity of the functions

i) $w = 2x - 3y + i(3x + 2y)$ (4 marks)

ii) $f(z) = |z|^2$ (3 marks)

b. Construct an analytic c function whose real part $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + c$ (5marks)

c. Derive the Cauchy Riemann equations in rectangular coordinates (5 marks)

d. Evaluate the integral $\oint_c \frac{z^2+9}{z-3}\, dz$ (3 marks)

QUESTION THREE (20MARKS)

a. Given that $\frac{5}{x+iy} + \frac{2}{1+3i} = 1$ where x and y are real numbers, find their values.

(4 marks)

b. Consider the function $f(z) = (2y - x) + i(x^2 - y)$. Determine the value of x that will make the function analytic. (5 marks)

c. Find the Laurent series about the indicated singularity for each of the following functions and give the region of convergence of each series

i) $\frac{e^{2z}}{(z-1)^3}; z = 1$ (6 marks)

ii) $\frac{z - \sin z}{z^3}; z = 0$ (5 marks)

QUESTION FOUR (20MARKS)

a. State the Cauchy's integral formula and hence evaluate $\frac{1}{2\pi i}\oint_c \frac{\cos \pi z}{z^2 - 1} dz$ where c is a rectangle with vertices $-i, -2-i, -2+i, i$ (7 marks)

b. Evaluate the following residues

i) $\underset{z=0}{\operatorname{Res}} \left(\frac{\sin z}{z^3} \right)$ (3 marks)

ii) $\underset{z=0}{\operatorname{Res}} \left(z e^{\frac{1}{z}} \right)$ (3 marks)

iii) Res $f(z)$; $f(z) = \frac{z+2}{z^2(z+1)}$ (7 marks)

QUESTION FIVE (20MARKS)

a. Expand $f(z) = \frac{1}{z-1}$ as a Taylor series about $z = 0$ (4marks)

b. State the Cauchy's residue theorem and use it to evaluate $I = \frac{1}{2\pi i}\int_c \frac{z+2}{z^2(z+1)}\,dz$, where c is the circle $|z| = 2$ (10 marks)

c. Evaluate the integral $\frac{1}{2\pi i}\oint \frac{z^2+5}{z-2}\,dz$ (6 marks)