Quantitative Techniques

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REFERENCES

We gratefully acknowledge use of the Kenya Accountants and Secretaries National Examination Board (Kasneb). Other reference books include

- Principles of operations research for management by F S. Budnick,
- D. Mcleavey, R. Mojena 2nd edition
- Quantitative techniques simplified by N.A Saleemi
- Quantitave techniques for managerial decision by U K Srivastara, G R Shenoy and S C Sharma
- Quantitative techniques by T Lucey 4th edition

Part I: Introduction

How To Use This Revision Kit

Questions and answers have been presented following from the structure of the quantitative techniques paper. Looking at the chapter arrangement the structure can be seen. After getting the concept from the Distance Learning Course Package (DLC package) and trying the exercises at the end of the topics, you can now attempt the questions from the revision kit.

Questions have been picked from various books before questions from past papers. Answer progressively to the questions from the past papers. In this way you will have been introduced to exam situation.

Do not look at the answer while answering the question. First attempt the question, and then look at the answer. Where you have gone wrong is where you need to work on. Read the model answers and get more information from the DLC package and the books recommended. As you progress in answering, time yourself to work in the shortest time possible remembering the exam techniques highlighted later (formulas, explanations of steps, number of significant figures).

At the end, there are three mock papers. Do at least one paper in exam situation (timing yourself for three hours continuously). If possible do all the questions in 5 hours. This will ensure you have grasped all the concepts of quantitative techniques.

Then mark for yourself using the model answers.
If possible do all the mock papers and mark for yourself. This will give you leverage on what to expect in exams.

General Examination Techniques

Structure Of The Paper

The paper consist of two sections **Section 1** and **Section 2**

- **Section 1** has five questions and usually you are required to answer any 3.The section mostly covers the whole syllabus but does not concentrate on operations research. Every question contains both mathematical and essay parts.
- **Section 2** has three questions and usually you are required to answer any 2. The section mainly covers operations research but can come from any topic. Questions go more into detail on a specific topic. The areas mostly covered here are network analysis, decision analysis/game theory and linear programming. In depth knowledge of a particular topic is required

General Trend

The examiner has been deviating slightly from common questions of mathematical nature. Knowledge of the topics is necessary other than just the mechanics of the calculations alone.
The examiner wants you to know more about the little topics that are covered like transportation and assignment, time series and smoothening.

How To Tackle Questions

- Follow the instructions (Complete the number of questions required in each section without answering more than required)
- Always start with questions that are easy for you.
- Be time conscious. Do not spend more than 1.8 minutes (1 minute 48 seconds) per mark awarded.
- Be careful on things like number of significant figures and decimal points. (Not giving too many or too few depending on the question)
- Write the formula that you are to use before actually using it.
- Try as much as possible to explain the calculation as you go through it step by step
- Answer what is required and not what you know about a particular subject.

Tit Bit

¾ Quantitative techniques subject has several equations/formulas. These are to be memorized. To make it easy, you are requested to do as many exercises as possible to enable ease of remembrance of the formulas and steps.

¾ Definitions, advantages, disadvantages and area of use of the various formulas and theories have to be known. This is there for every question

Syllabus

CPA PART II - SECTION 4

PAPER NO.11 QUANTITATIVE TECHNIQUES

OBJECTIVE
To provide the candidate with quantitative techniques for use in solving managerial problems.

11.0 **SPECIFIC OBJECTIVES**

A candidate who passes this subject should be able to:

- Apply quantitative methods in solving optimisation problems in management.
- Use statistical methods in decision making.
- Design statistical and mathematical models for estimation and forecasting.
- Use network analysis in project management.

CONTENT

11.1 **Linear Algebra and Calculus**

- Sets and set theory
- Functions and graphs
- Linear equations, higher-order equations, inequalities and simultaneous equations.
- Matrix algebra
- Application of matrix algebra to input-output analysis and elementary makovian processes
- Differentiation and integration of polynomial, exponential and logarithmic functions.
- Application of calculus to economic models.

11.2 **Descriptive Statistics and Index Numbers**

- Measures of location
- Measures of dispersion
- Bivariate data: Regression analysis, covariance and correlation, coefficient of determination, estimation and forecasting
- Multivariate data: Multiple linear regression, least squares method, coefficient of correlation and determination, estimation and forecasting.
- Unregistered price and quantity indices.
- Weighted, index numbers
- Fixed base and change of base period; chain base indices
- Tests of perfection; consumer price index, stock market index and production indices.

11.3 **Probability**

- Definition of probability - relative frequency approach, sample space
- Events; mutually exclusive events and independent events
- Laws of probability, conditional probability and Bays theorem
- Permutations and combinations
- Discrete probability distribution functions; Binomial multinomial, hyper-geometric, geometric and Poisson distribution
  Continuous probability distribution functions: Exponential, normal, student -t, chi-square and fisher distributions

11.4 **Sampling and Estimation**

- Random, cluster, stratified and systematic sampling
- The central limit theorem
- Sampling distribution of the mean and difference between means
- Sampling distribution of proportions and differences between proportions
- Tests of hypothesis

11.5 **Analysis of Time Series Analysis**

- Time series models, trends, seasonal variations and

forecasting. 11.6 **Linear Programming**

- Definition of decision variables, objective and constraint functions
- Formulation of a linear programming problem, graphical solution, introduction to simplex method, interpretation of computer assisted solution printout.
- Duality: Economic meaning of shadow prices and other computer output data
- An initiative (but not computational) understanding of integer, dynamic and convex programming techniques
- Application to transport and assignment problems.

11.7 **Decision Theory**

- Decision making under conditions of risk and uncertainty
- The expected value criterion, the minmax and maxmax criteria, conditional payoff and opportunity cost tables
- Decision trees and sequential decisions
- Game theory

11.8 **Network Analysis**

- Activities sequencing and Gantt Charts
- Identification of critical path
- Floats and non-critical activities
- Crashing of projects
- Stochastic critical path analysis
- Cost and Resources scheduling

Topical Guide to Past Paper Questions

Chapters arranged according to exam question flow

Topic 1	Linear algebra, matrices algebra application, calculus and set theory
Topic 2	Measures of central tendencies, measures of dispersion and index numbers
Topic 3	Basic probability and probability distributions
Topic 4	Sampling and sampling inference
Topic 5	Measures of relationship, linear regression, correlation and time series analysis
Topic 6	Linear programming
Topic 7	Decision theory and game theory
Topic 8	Network analysis
Topic 9	Mock paper and model answers

Part II: Revision Questions and Answers

QUESTIONS - PAST PAPERS

LINEAR ALGEBRA & CALCULUS

QUESTION ONE

A company is selling an item for p_s= Sh.5. Profit is related to selling price p by

$$P = 50000\, ps - 6250\, ps^{2}$$

i) Is the profit an increasing function or a decreasing function when p_s = 5?
ii) If the price is changed to Sh.4.75, find whether profit would increase or decrease and find the change in profit.
iii) At what price is the profit maximized?

QUESTION TWO

Demand function for a firm is given by

$$P = 12 - 0.4Q$$

P is the price of the product, Q is the quantity demanded, and the total cost (C) is given by

$$C = 5+4Q+0.6Q^2$$

At what price and quantity will the firm have maximum profit? If the firm aims at maximizing sales, what price should it charge?

QUESTION THREE

a) Two CPA students were discussing the relationship between average cost and total cost. One student said that since average cost is obtained by dividing the cost function by the number of units Q, it follows that the derivative of the average cost is the same as marginal cost, since the derivative of Q is 1.

Required:
Comment on this analysis.

b) Gatheru and Kabiru Certified Public Accountants have recently started to give business advise to their clients. Acting as consultants, they have estimated the demand curve of a clients firm to be;

AR=200-8Q

Where AR is average revenue in millions of shillings and Q is the output in units.
Investigation of the client firm's cost profile shows that marginal cost (MC) is given by:

$MC=Q^2-28Q+211$(In million shillings)

Further investigations have shown that the firm's cost when not producing output is sh.10 million.

Required:
i) The equation of total cost
ii) The equation of total revenue
iii) An expression for profit.
iv) The level of output that maximizes profit
v) The equation of marginal revenue.

(Q 1 June 2002)

QUESTION FOUR

XYZ Company Limited invests in a particular project and it has been estimated that after X months of running, the cumulative profit (Sh.'000') from the project is given by the function $10x - x^2 - 5$, where x represents time in months. The project can run for eleven months at most.

Required:

i) Determine the initial cost of the project.
ii) Calculate the break-even time in months for the project.
iii) Determine the best time to end the project.
iv) Determine the total profit within the break-even points.

(Q 1 Dec 2001(b)

QUESTION FIVE

a) The number of shoppers queuing at any given time in a certain supermarket in downtown Nairobi can be approximately represented by the equation:
$y = x^3 - 14x^2 + 50x$ over the range $0 \leq x \leq 8.5$, where y is the number queuing and x is the time in hours after the store opens at 9.00a.m. (So that, for example 10.30a.m. is x=1.5, and 5.30p.m. - when the store closes is x= 8.5).

Required:

i) The management wants to know when they should deploy more cashiers and the number queuing at that time.
ii) Determine the number of man-hours spent per day by shoppers queuing.

b) An electronics firm carries out a small-scale test launch of a new low-priced pocket calculator. It estimates from this test that if it went into full-scale production it would sell between 1,000 and 2,500 calculators per month, and that its monthly revenue in thousands of shillings over this range of sales could be represented by the equation:
$R = - x^2 + 5x$

Where: x is the monthly output in thousands of calculators (it is assumed that it sells its entire output).

From experience of calculator production, the firm estimates its marginal cost in thousands of shillings could be represented by the equation:
$MC = x^2 - x + 2$
and that its fixed costs will be Sh.500 per month.

Required:

i) Determine the average cost and revenue equations for this firm.
ii) Determine the profit-maximizing output, the price that should be charged to maximize profit, and how much each calculator will then cost to make.

(Q 1 June 2001)

QUESTION SIX

a) Define the following terms as used in Markovian analysis:
i) Transition matrix.
ii) Initial Probability vector
iii) Equilibrium
iv) Absorbing state

b) A company employs four classes of machine operators (A,B,C,D): all new employees are hired as class D and, through a system of promotion, may work up to a higher class. Currently, there are 200 class D, 150 class C, 90 class B and 60 class A employees. The company has signed an agreement with the union specifying that 20 percent of all employees in each class be promoted, one class in each year. Statistics show that each year 25 percent of the class D employees are separated from the company by reason such as retirement, resignation and death. Similarly 15 percent of class C, 10 percent of class B and 5 percent of class A employees are also separated. For each employee lost, the company hires a new class D employee.

Required:
i) The transition matrix.
ii) The number of employees in each class two years after the agreement with the union.
iii) The equilibrium state in number of employees.

(Q 2 June 2002)

QUESTION SEVEN

a) Two firms A and B in Nairobi Industrial area make glue. The cost functions for making glue for the two firms are as follows:
Firm A - $C = 0.2x + 200$
Firm B - $C = 0.6x + 50$

Where x – is litres of glue produced in '000'.

Required:
By drawing a graph of these functions, show whose firm's costs increase more rapidly.

b) On the study of costs and revenue for production of biro pens by a small company ACO Ltd, the following expressions were determined. Before production starts a set-up cost of Sh.1500 existed.

$AR = 600 – 0.5q$ Average revenue
$MC = 140 – 8q + 0.15q^2$ Marginal cost

Required:
i) The expression for profit.
ii) At what quantity q is profit maximized.
iii) Is this quantity q also the point that revenue is maximized?

QUESTION EIGHT

a) Explain the following terms as used in calculus:
i) Turning point.
ii) Second order derivative condition.
iii) Partial derivative.
iv) Mixed partial derivative.
v) Saddle point.

b) Drumstick Chicken Wings Ltd supplies chicken wings for Kuku Inn with the following demand and cost functions for a given week:

$P = 100 – 0.01x$ - Price
$TC = 50x + 30,000$ - Total cost

Where:
x – number of chicken wings supplied.

Required:
i) Total revenue for Drumstick Chicken Wings Ltd.
ii) Determine the number of chicken wings that maximize weekly profit.
iii) What is the difference in profit if the Drumstick Chicken Wings Ltd. objective is to maximize revenue rather than profit?

QUESTION NINE

a) Given the following input – output matrix and demand vector of shoes S, rubber R and glue G industries, determine the production vector.

Input – output matrix

$$\begin{array}{c|ccc} & S & R & G \\ \hline S & 0.3 & 0.2 & 0.1 \\ R & 0.1 & 0.4 & 0.2 \\ G & 0.2 & 0.3 & 0.4 \end{array}$$

Demand vector

$$\begin{array}{c} 40 \\ 50 \\ 60 \end{array}$$

b) If in (a) above the demand of industries changes as follows:

S decreases by 10 units
R increases by 5 units
C increases by 10 units.

What should be the production levels?

DESCRIPTIVE STATISTICS AND INDEX NUMBERS

QUESTION ONE

a) The index of industrial production in the Utopia country by July 2001 is given below:

Sector	Weight	July 2001 Index (1994 = 100)
Mining and quarrying	41	361
Manufacturing:		
Food, drink and tobacco	77	106
Chemicals	66	109
Metal	47	72
Engineering	298	86
Textiles	67	70
Other manufacturing	142	91
Construction	182	84
Gas, electricity and water	80	115

Required:

i) Calculate the index of industrial production for all industries and manufacturing industries.
ii) Comment on your results.

b) Explain some of the uses of index numbers.

c) What are some of the limitations of index numbers?

(Q 2 Dec 2001)

QUESTION TWO

a) The table below shows the income per month of borrowers and the percentage of all mortgages as provided by Building Society Mortgages in the year 2000.

Income per month of borrowers (Sh.)	Percentage of all mortgages
Under 30,000	5
30,000 - 34,999	2
35,000 - 39,999	3
40,000 - 44,999	5
45,000 - 49,999	10
50,000 - 59,999	15
60,000 - 69,999	18
70,000 - 99,999	21
100,000 - 149,999	17
150,000 and over	4

Required:

i) Calculate some suitable measures of central tendency and dispersion and comment on your results.
ii) Explain with an example the value of descriptive statistics in the accounting function.

(Q 3 (a) Dec 2001)

QUESTION THREE

a) A machine produces circular bolts and, as a quality control test, 250 bolts were selected randomly and the diameter of their heads measured as follows:

Diameter of head (cm)	Number of components
0.9747 - 0.9749	2
0.9750 - 0.9752	6
0.9753 - 0.9755	8
0.9756 - 0.9758	15
0.9759 - 0.9761	42
0.9762 - 0.9764	68
0.9765 - 0.9767	49
0.9768 - 0.9770	25
0.9771 - 0.9773	18
0.9774 - 0.9776	12
0.9777 - 0.9779	4
0.9780 - 0.9782	1

Required:
Determine whether the customer is getting reasonable value if the label on the circular bolt advertises that the average diameter of the head is 0.97642 cm.

b)
i) In what situation would a weighted mean be used?
ii) Describe briefly how to estimate the median on a grouped frequency distribution graphically.
iii) Why is the mode not used extensively in statistical analysis?
iv) " The standard deviation is the natural partner to the mean". Explain.

(Q 2 June 2001)

QUESTION FOUR

a) Explain the following terms as used in index numbers:
i) Quantity indices.
ii) Base year.
iii) Chain index numbers.
iv) Retail price index.

b) The table below reports the annual net salary amounts for an accountant who begun employment with an auditing firm in 1995. In addition, the values of the Consumer Price Index (CPI) for 1995 through 1999 is shown.

Year	Salary (Sh.)	CPI
1995	360,000	130.7
1996	370,000	136.2
1997	390,000	140.3
1998	395,000	144.5
1999	400,000	148.2

Required:
Determine whether there is a difference in percentage salary increase between 1995 and 1999 in terms of stated (current) shillings amounts and in terms of constant (deflated salary, using CPI) shillings.

(Q 2 Dec 2000)

QUESTION FIVE

Moving averages are often used in an effort to identify movements in share prices. Approximate monthly closing prices (in Sh. per share) for Toys Children Ltd. for December 2000 through November 2001 are shown below:

Month		Price (Sh.)
December	2000	40
January	2001	38
February	2001	39
March	2001	41
April	2001	36
May	2001	41
June	2001	34
July	2001	37
August	2001	35
September	2001	37
October	2001	40
November	2001	41

Required:

a) Use a 3-month moving average to forecast the closing price for December 2001.

b) Use a 3-month weighted moving average to forecast the closing price for December 2001. Use weights of 0.4 for most recent period, 0.4 for the second period back and 0.2 for the third period back.

c) Use exponential smoothing constant of $\alpha = 0.35$ to forecast the closing price for December 2001.

d) Which of the three methods do you prefer? Why?

(Q 5 Dec 2001)

PROBABILITY

QUESTION ONE
A problem is given to three managers A, B, C whose chances of solving are ½, ⅓, ¼ respectively. What is the probability that the problem will be solved?

QUESTION TWO
Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1girl and 3 boys. One child is selected at random from each group, show that the chance that the three selected, consist of 1 girl and 2 boys is 13/32.

QUESTION THREE
The following table gives a bi-variate frequency distribution of 50 managers according to their age and salary (in rupees).

Salary in rupees

Age in years	1000-1500	1500-2000	2000-2500	2500-3000	Total
20-30	2	3	-	-	5
30-40	5	4	2	1	12
40-50	-	2	10	3	15
50-60	-	1	8	9	18
Total	7	10	20	13	50

If a manager is chosen at random from the above distribution, find the chance that; (i) he is in the age group of 30-40 and earns more than Rs.1500, (ii) his earnings are in the range of Rs.2000-2500 and is less than 50 years old.

QUESTION FOUR
Computer analysis of satellite data has correctly forecast locations of economic oil deposits 80% of the time. The last 24 oil wells drilled produced only 8 wells that were economic. The latest analysis indicates economic quantities at a particular location. What is the probability that the well will produce economic quantities of oil?

QUESTION FIVE
A firm recently submitted a bid for a turnkey project for a 500 MW power plant. If its main competitor submits a bid, the chances of bid being awarded to the firm is 0.3. If the main competitor doesn't bid, there is a ¾ chance of the firm getting the contract. There is a 0.50 chance that the main competitor will bid.
i) What is the probability of the firm's getting the contract?
ii) What is the probability that the competitor's bid given that the firm's bid is awarded?

QUESTION SIX
A firm has four plants scattered around the city producing the same homogeneous item at all plants. The first plant produces 30 per cent of the total production, second plant 25 per cent, third plant 35 per cent and the fourth plant 10 per cent. The firm has a single warehouse in the city for storing the finished product of all the plants without any distinction. From the past performance records on the proportion of defectives, it has been found that 5 per cent, 10 per cent, 15 per cent and 20 per cent from the items produced at plants 1, 2, 3, and 4 respectively are defective. Before the shipment of the items to a dealer, one unit is selected and found defective. What is the probability that the item was produced in plant 3?

QUESTION SEVEN
a) State clearly what is meant by two events being statistically independent..
b) In a certain factory that employs 500 men, 2% of all employees have a minor accident in a given year. Of these, 30% had safety instructions whereas 80% of all employees had no safety instructions.
   Required:
   Find the probability of an employee being accident-free given that he had:
   i) no safety instructions.
   ii) safety instructions.

c) An electric utility company has found out that the weekly number of occurrences of lightning striking the transformers is a Poisson distribution with mean 0.4.

Required:
i) The probability that no transformer will be struck in a week.
ii) The probability that at most two transformers will be struck in a week.

(Q 3 June 2002)

QUESTION EIGHT

a) The past records of Salama Industries indicate that 4 out of 10 of the company's orders are for export. Further, their records indicate that 48 percent of all orders are for export in one particular quarter. They expect to satisfy about 80 orders in the next financial quarter.

Required:
i) Determine the probability that they will break their previous export record
ii) Explain why you have used the approach you have chosen to solve part (i) above.

b) Grear Tyre Company has just developed a new steel-belted radial tyre that will be sold through a chain of discount stores. Because the tyre is a new product, the company's management believes that the mileage guarantee offered with the tyre will be an important factor in the consumer acceptance of product. Before finalizing the tyre mileage guarantee policy, the actual road test with the tyres shows that the mean tyre mileage is $\mu = 36{,}500$ kilometres and the standard deviation is σ =5,000 kilometres. In addition, the data collected indicate that a normal distribution is a reasonable assumption.

Required:
i) Grear Tyre Company will distribute the tyres if 20 per cent of the tyres manufactured can be expected to last more than 40,000 kilometers. Should the company distribute the tyres?

ii) The company will provide a discount on a new set of tyres if the mileage on the original tyres does not exceed the mileage stated on the guarantee.

What should the guarantee mileage be if the company wants no more than 10% of the tyres to be eligible for the discount?

c) Explain briefly some of the advantages of the standard normal distribution.

(Q 4 Dec 2001)

QUESTION NINE

a) In a particular life insurance office, employees Simiyu, Juki, Waithera and Baraza have a diploma, with Simiyu and Baraza also having a degree. Simiyu, Macharia, Waithera, Thuo, Mwanzia and Kungu are associate members of the Chartered Insurance Institute (ACII) with Thuo and Mwanzia having a diploma.

Required:
Identifying set A as those employees with diploma, set C as those employees who are ACII and set D as having a degree:
i) Specify the elements of sets A, C and D.
ii) Draw a Venn diagram representing sets A, C and D, together with their known elements.
iii) What special relationship exists between sets A and D?
iv) Specify the elements of the following sets and for each set, state in words what information is being conveyed: $A \cap C$; $D \cup C$ and $D \cap C$.
v) What will be a suitable universal set for this situation?

b) The purchasing department has analysed the number of orders placed by each of the 5 departments in the company by type of this financial year as given in the table below:

Order type	Sales	Purchase	Production	Accounts	Maintenance	Total
Consumables	10	12	4	8	4	38
Equipment	1	3	9	1	1	15
Special	0	0	4	1	2	7
Total	11	15	17	10	7	60

An error has been found in one of these orders.

Required:
i) Determine the probability that the incorrect order was not for consumables.
ii) Determine the probability that the incorrect order came from maintenance or production.
iii) Calculate the probability that the incorrect order was an equipment order from purchase.

c)
i) Under what conditions does P (A/B) = P (A)?
ii) What is the addition rule of probability and for what type of events is it valid?
iii) What is Bayes Theorem?

(Q 3 June 2001)

QUESTION TEN
In each of the following three situations, use binomial, poisson, or normal distribution depending on which is the most appropriate.

In each case, explain why you selected the distribution and draw attention to any feature which supports or casts doubt on the choice of distribution.

a) Situation 1:
The lifetimes of a certain type of electrical components are distributed with a mean of 800 hours and standard deviation of 160 hours.

Required:
i) Identify situation 1.

ii) If the manufacturer replaces all the components that fail before the guaranteed minimum lifetime of 600 hours, what percentage of the components have to be replaced?

iii) If the manufacturer wishes to replace only 1% of the components that have the shortest life, what value should be used as the guaranteed lifetime?

iv) What is the probability that the mean lifetime of a sample of 25 of these electrical components exceeds 850 hours?

b) Situation 2:

A green grocer buys peaches in large consignments directly from wholesaler. In view of the perishable nature of the commodity, the green grocer accepts that 15% of the supplied peaches will usually be unsaleable. As he cannot check all the peaches individually, he selects a single batch of 10 peaches on which to base his decision of whether to purchase a large consignment or not. If no more than two of these peaches are unsatisfactory, the green grocer purchases the consignment.

Required:

i) Identify situation 2.
ii) Determine the probability that under normal supply conditions, the consignment is purchased.

c) Situation 3:

Vehicles pass a certain point on a busy single-lane road at an average rate of two per 10 second interval.

Required:

i) Identify situation 3.
ii) Determine the probability that more than three cars pass this point during a 20 second interval.

(Q 3 July 2000 – Pilot Paper)

QUESTION ELEVEN

a) Define probability as used in Quantitative Techniques.
b) What is Bayes Theorem? Explain how Bayes Theorem can be utilized practically.
c) KK accounting firm has noticed that of the companies it audits, 85% show no inventory shortages, 10% show small inventory shortages and 5% show large inventory shortages. KK firm has devised a new accounting test for which it believes the following probabilities hold:

P (Company will pass test/no shortage) = 0.90
P (Company will pass test/small shortage) = 0.50
P (Company will pass test/large shortage) = 0.20

Required:

i) Determine the probability if a company being audited fails this test has large or small inventory shortage.

ii) If a company being audited passes this test, what is the probability of no inventory shortage?

(Q 7 June 2000)

SAMPLING & ESTIMATION

QUESTION ONE
Suppose it is known that the mean annual income of workers in a certain community is Rs.5,000 with a standard deviation of Rs.1,200. A researcher suspects the "son of soil" workers have higher than the average income. He draws a random sample of 144 local workers and obtains the sample mean of Rs.5,500. Can he say the local workers have significantly higher income than the total population? (Use $\alpha = 0.05$).

QUESTION TWO
The sales Manager of a large sales force is interested in finding whether or not average number of weekly sales contacts per sales representative (15) has changed. The sample information was collected and it was found that 200 sales representatives had an average of 16.5 contacts per week with a standard deviation of 3.5. At 5 per cent level of significance, what should the sales manager conclude?

QUESTION THREE
Two different models are available for the same machine. The production statistics (number of units produced per hour) of these two models are given below. The data was collected on different days.

Model A:	180,	176,	184,	181,	190,	137,	
Model B:	195,	194,	190,	192,	187,	185,	187,

Will you conclude that Model A and Model B have the same productivity?

QUESTION FOUR
The General Manager of a large hotel in Bombay wishes to know whether or not the personal services provided to its customers are uniform throughout the hotel industry. Whether the clients in the less expensive rooms get the same service as that of clients of more expensive rooms. The data collected from the comment cards is tabulated below:

	Room Charge			
	Rs	Rs	Rs	Rs
Rating	50	100	150	200
Excellent	10	25	28	28
Good	24	30	16	16
Average	80	82	21	13
Poor	40	41	18	25

Were the customers treated differently according to their room charges? Test the hypothesis at $\alpha = 0.05$ level of significance.

QUESTION FIVE
a) What is a one-sided confidence interval? When is it necessary?

b) What is wrong with the following hypotheses?

Null: $\bar{x} \leq 4.7$hours

Alternative: $\bar{x} > 4.7$hours

c) If a Type II error is costly but a Type I error is not, why should you set the level of significance α at 0.10, 0.20 or even higher?

d) Two researchers in marketing, Catherine Mbugua and George Otieno, reviewed the theory of product life cycle with respect to a specific product. They conducted a research project to determine the applicability of the theory to popular music records. As part of the study, they collected extensive information on a sample of 12 music records judged on the basis of certain well-defined criteria to be successes and a sample of 10 music records to be failures. They collected data on each music record for a period of 16 weeks from the date the record was released to the market. One item of information they collected on each music record was radio airplay. Measurement of this variable yielded the following means and standard deviations for the two samples of music records.

	$\bar{x}$	Sd
Successes	16	2.32
Failures	4	2.18

Required:

i) Can one conclude on the basis of these data that successful and unsuccessful (failures) differ with respect to mean amount of airplay? Let $\alpha = 0.01$

Note: Test statistic is distributed as student's t and is given by:

$$t = \frac{(\bar{x}_s - \bar{x}_f) - (\mu_s - \mu_f)}{\left(\frac{s^2 p}{n_1} + \frac{s^2 p}{n_2}\right)^{\frac{1}{2}}}$$

Where $\bar{x}_s$ = mean success

$\bar{x}_f$ = mean failures

$s^2 p$ = common population variance

$$S^2p = \frac{(n_1 - 1)Sd^2s + (n_2 - 1)Sd^2f}{n_1 + n_2 - 2}$$

Sd_s = Standard deviation success. And

Sd_f = Standard deviation failures.

ii) What assumptions are we making in order to use the pooled estimate of the common population variance?

Q 4 June 1999)

QUESTION SIX

a) Prior to an advertising campaign, 35 per cent of a sample of 400 housewives used a certain product. After the campaign, 40 per cent of a second sample of 400 housewives used the same product.

Required:

Did the campaign increase sales?

b) A manager is convinced that a new type of machine does not affect production at the company's major shop floor. In order to test this, 12 samples of this week's hourly output is taken and the average production per hour is measured as 1158 with a standard deviation of 71. The output per hour averaged 1196 before the machine was introduced.

Required:

Test the manager's conviction

c) i) Give some examples of the type of data that form a normal distribution.

ii) Under what conditions can the normal distribution be used as an approximation to the binomial distribution?

iii) What do confidence limits measure?

(Q 4 June 2001)

QUESTION SEVEN

a) Mia Shillings Department Stores is planning to open a store in Westlands. It has asked the Superior Marketing Company (SMC) to do a market study of randomly selected families within a five-kilometer radius of the store. Among the questions it wishes SMC to ask each home owner are:
   i) family income,
   ii) family size,
   iii) distance from home to the store site, and
   iv) whether the family owns a dog or a cat.

   Required:
   For each of the four questions, develop a random variable of interest to Mia Shillings Department Stores. Denote which of these are discrete and which are continuous random variables.

b) Mia Shillings Department Stores has compiled the following data concerning its daily sales. The sales of each day of the week are normally distributed with the following parameters:

Day	Mean (μ) in Sh.	Standard deviations (σ) in Sh.
Monday	120,000	20,000
Tuesday	100,000	25,000
Wednesday	100,000	10,000
Thursday	120,000	40,000
Friday	140,000	20,000
Saturday	160,000	50,000

   Required:
   Which day of the week has the lowest probability that the store will sell between Sh.110,000 and Sh.150,000 worth of goods?

c) Because of economic conditions, a firm reports that 30 per cent of its accounts receivable from other business firms are overdue. The firm's policy is that if an accountant takes a random sample of five such accounts and finds that exactly 20 per cent of the accounts are overdue 10 per cent of the time, then a warning letter should be written to the particular firm.

   Required:
   Should the warning letter be written?

(Q 3 Dec 2000)

ANALYSIS & TIME SERIES ANALYSIS

QUESTION ONE

Unlisted plc hopes to achieve a Stock Market quotation for its shares. A profit forecast is necessary and, in order to achieve such a forecast, the company has experimented with a number of approaches. The following are details from a linear regression on the last 11 years' profit figures:

x = years (expressed 1to 11)
y = annual profit figures

$$\sum x = 66$$
$$\sum y = 212.10$$
$$\sum x^2 = 506$$
$$\sum xy = 1{,}406.70$$
$$\sum y^2 = 4{,}254.08$$

$\sum (y - \hat{y})^2 = 0.916$ where $\hat{y}$ represents profit values estimated by the regression line.

The following formulae are given:

$$\text{Standard error of the regression line } \sigma_R = \sqrt{\frac{\sum (y - \hat{y})^2}{df}}$$

$$\text{Coefficient of correlation (r)} = \sqrt{\frac{\textit{Explained variation}}{\textit{Total variation}}}$$

You are required:

a) To obtain the simple least squares regression line of Y on X;
b) To use the line to estimate profit in each of the next two years;
c) To calculate the coefficient of determination for the line and to explain its meaning;
d) To calculate the standard error of the regression line and to use this to obtain the 95% confidence interval for the line;
e) On the basis of the information given on your answer (a) to (d) to determine whether it is likely that the regression line will be a good estimator of profit.

QUESTION TWO

The following data have been collected relating to returns which would have been earned from an investment of an equal sum of money in the shares of E.T. plc and a group of market shares:

Year	Returns on E.T. plc shares (y) (£)	Returns of market shares (x) (£)
1	7.8	11.1
2	11.0	12.3
3	15.2	18.5
4	23.1	25.4
5	29.7	28.7
6	37.4	33.8
7	44.6	37.7
8	52.8	39.6
9	60.2	44.7
10	63.9	45.5

$\sum y = 345.7$, $\sum X = 297.3$, $\sum y^2 = 15{,}711.59$, $\sum x^2 = 10{,}285.83$, $\sum xy = 12{,}577.02$

The standard error of the regression coefficient is:

Standard error of the regression line

$$\sqrt{\sum x_i^2 - n\bar{x}^2}$$

and where the standard error of the regression line is:

$$\sqrt{\frac{\sum (y_i - \hat{y})^2}{n-2}}$$

You are required to:

a) Plot the original data on a graph, calculate the least squares regression of y on x and draw the regression line on the graph;
b) Test the significance of the slope of the line at the 5% level;
c) Discuss the outcomes of (a) and (b) in the context of a comparison of the two investments

QUESTION THREE

The following regression equation was calculated for class of 24 CPA II students. -

$\hat{y} = 3.1 + 0.021x_1 + 0.075x_2 + 0.043x_3$

Standard error (0.0190) (0.034) (0.018)

Where y=students score on a theory examination
x_1 = Students rank (from the bottom) in high school
x_2 = Students verbal aptitude score
x_3 =A measure of students character

Required:

a) Calculate the t ratio and the 95% confidence interval for each regression coefficient.
b) What assumptions did you make in (a) above? How reasonable are they?
c) Which regressor gives the strongest evidence of being statistically discernible?
d) In writing up a final regression equation, should one keep the first regressor in the equation, or drop if? Why?

(Q 5 June 2002)

QUESTION FOUR

a) Does finding a no linear relationship between two variables mean no relationship?
b) Does a high correlation mean that one variable causes another variable to vary?
c) In recent years environmentalists and health professionals have been concerned about the ill effects on the environment of the widespread use of insecticides. If human beings are to cope with the problem and make decisions about how to deal with it, they must understand the effect of insecticides on humans and other animals. In the kind of study often done to promote such understanding, two professors at University of Nairobi investigated the effect of a commonly used insecticide on sheep. Among other statistical analyses, they derived the following linear regression equation (n = 16):

   $\hat{y} = 27.32 + 1.3x$

   This equation describes the relationship between the activity of a certain enzyme in the sheep's brain (Y) and the time (in hours) after the sheep has been exposed to the insecticide (X).

   Required:

   i) Suppose 30 hours have elapsed since a sheep has been exposed to the insecticide, what is the predicted value of Y?
   ii) How would you describe the relationship between the two variables?
   iii) The professors computed a coefficient of determination (r^2) of 0.86 from the data. What conclusion can be drawn about the true relationship between the two variables? Let $\alpha = 0.05$
   iv) What assumption are necessary to solve part (iii) above?

(Q 5 June 1999)

QUESTION FIVE

Kenya Graduate School (KGS) offers a variety of graduate courses. However, its main emphasis has been on information science (IS) courses. Due to the laboratory equipment requirements for IS courses, KGS has to estimate in advance the expected students enrolments. Over the last 5 years, the students enrolments, by quarter, has been:

Years

Quarter	*1991*	*1992*	*1993*	*1994*	*1995*
First	30	32	41	45	73
Second	42	107	93	101	181
Third	100	71	139	151	227
Fourth	66	47	62	67	109

Required:

a) Determine the estimates, by quarter, for year 1996. Justify the method you use.
b) If linear multiple regression were to be used in order to determine the predicting equation, what other variables would be included?
c) How would the expected enrolments be compared to the actual enrolments?

Note:

$\sum x = 210 \qquad \sum y = 1,784$

$\sum x^2 = 2,870 \qquad \sum xy = 22,253$

(Q 4 Dec 1995)

QUESTION SIX

a) Briefly but clearly, explain the difference, if any, between regression analysis and correlation analysis.
b) As a tax consultant to the Government of your country, you have been asked to estimate an empirical model on the demand for income tax evasion (ITE) in the country. You think true income (TI), Marginal tax rate (MTR), penalty rate (PR) and probability of detection (PROB) will be important variables. Using national time-series annual data from 1967 to 1995, you estimate the following regression equation:

$$ITE_t = -52.59 + 33.44 \log TI_t + 0.93MTR_t - 0.20PR_t - 1.48PROB_t$$
$$(-6.85) \quad (13.97) \quad (6.35) \quad (-2.93) \quad (-3.90)$$

The calculated t – statistics are reported in parentheses, and figures are in shillings.

Required:

i) Write down the population theoretical empirical model on the demand for income tax evasion (ITE).
ii) Provide a theoretical justification of the empirical model specification in (i) above, that is, expected signs of the regression coefficients and why.
iii) Interpret the constant term (-52.59), coefficient for true income (33.44) and coefficient of probability of detection (-1.48) in the context of the problem.
iv) Before collecting data, the principal tax collector and her staff believed that penalty rate had a negative influence on income tax evasion and should therefore be used as leverage on those who evade tax.
From the regression results, should this be the case? Why?
(Note: critical t-value = - 1.701 at 0.05 level of significance)

(Q 5 June 1996)

QUESTION SEVEN

Find the moving average of the time series of quarterly production (in tons) of coffee in an Indian State as given below. After that, come up with a trend line to approximate the production in future.

Production (in Tons)

Year	Quarter I	Quarter II	Quarter III	Quarter IV
1983	-	-	12	16
1984	5	1	10	17
1985	7	1	10	16
1986	9	3	8	18
1987	5	2	15	5

QUESTION EIGHT

a) The number of plumbing repair jobs by Manji Plumbing Service in each of the last nine months in Nakuru town are listed below:

Month	**Jobs**
March	353
April	387
May	342
June	374
July	396
August	409
September	399
October	412
November	408

Required:

a)

i) Forecast the number of repair jobs Manji Plumbing Service will perform in December. Use the least squares method.

Note:

Slope = $\left(n \sum tY_t - \sum t \sum Y_t\right) / \, n \sum t_2 - \left(\sum t\right)^2$

And

Intercept = Y – b_1t

ii) What is your forecast for December using a three-period weighted moving average with weights of 0.6, 0.3, and 0.1. How does it compare with your forecast from part (i) above?

b)

i) What are the aims of time series analysis?

ii) Describe what a season is in the context of a time series and give some examples.

iii) Describe the stages in obtaining a time series trend using the method of semi averages.

iv) Why must forecasts be treated with caution?

(Q 5 June 2001)

LINEAR PROGRAMMING

QUESTION ONE

a) Solve the following problem using the simplex method: Maximize

$$z = 7x_1 + 5x_2$$

Subject to

$$5x_1 + 3x_2 \leq 50 \quad (1)$$
$$4x_1 - 2x_2 \leq 30 \quad (2)$$
$$x_1, x_2 \geq 0.$$

b) Verify your solution by solving graphically and relate tableaux to specific corner points.

c) Determine the shadow prices and interpret their meaning.

QUESTION TWO

A company wishes to purchase additional machinery in a capital expansion program. Three types of machines are to be purchased: A, B, and C. Machine A costs $25,000 and requires 200 square feet of floor space for its operation. Machine B costs $ 30,000 and requires 250 square feet of floor space. Machine C costs $22,000 and requires 175 square feet of floor space. The total budget for this expansion program is $350,000. The maximum available floor space for the new machines is 4,000 square feet. The company also wishes to purchase at least one of each machine.

Given that machines A, B, and C can produce 250, 260, and 225 pieces per day, the company wants to determine how many machines of each type it should purchase so as to maximize daily output (in units) from the new machines.

a) Explicitly define your decision variables and formulate the LP model.

b) Assess the validity of the four underlying LP assumptions for this problem.

c) Solve and analyse the problem using a computer package

QUESTION THREE

A pension fund wishes to invest in one or more of six possible investments. Financial analysts have estimated the present value of effective annual estimate. The data in this table indicate that the present value of investing $10,000 in alternative 1 is the sum of $1,200 (0.12×$10,000) for year 1, $1,000 (0.10×10,000) for year 2, and $800 (0.08×$10,0000 for year 3, for a total present value of $3,000.

Effective Annual Rate of return

Investment	**Year 1**	**Year 2**	**Year 3**
1	0.12	0.10	0.08
2	0.14	0.10	0.10
3	0.15	0.12	0.08
4	0.10	0.12	0.15
5	0.08	0.12	0.18
6	0.25	0.15	0.05

Management has decided that $300,000 will be invested. At least $50,000 is to be invested in alternative 2 and no more than $40,000 in alternative 5. Total investment in alternative 4 and 6 should not exceed $75,000, as these are risky investments.

If the objective is to maximize the present value of total dollar return for the three-year period, formulate the LP model for how much capital to invest in each alternative.

Can you comment on how relevant each underlying LP assumption is to this problem?

QUESTION FOUR

An endowment fund manager is attempting to determine a "best" investment portfolio and is considering six alternative investments. The following table indicates point estimates for the price per share, the annual growth rate in the price per share, the annual dividend per share, and a measure of the risk associated with each investment.

Portfolio data

	Alternative					
	1	2	3	4	5	6
Current price per share	$80	$100	$160	$120	$150	$200
Projected annual growth rate	0.08	0.07	0.10	0.12	0.09	0.15
Projected annual dividend per share	$4.00	$6.50	$1.00	$0.50	$2.75	0
Projected risk	0.05	0.03	0.10	0.20	0.06	0.08

In this case risk is defined as the standard deviation in return. Dollar return per share of stock is defined as price per share one year hence less price per share plus dividend per share. The fund has $2.5 million to invest, and it wishes to satisfy the following conditions:

The maximum dollar amount to be invested in alternative 6 is $250,000
No more than $500,000 should be invested in alternatives 1 and 2, combined
Total weighted risk should be no greater than 0.10, where

$$\textit{Total weighted risk} = \sum \left[\frac{(\textit{Dollars invested in alternative } j) \times (\textit{Risk of alternative } j)}{\textit{Total dollars inveted in all alternatives}} \right]$$

For the sake of diversity, at least 100 shares of each stock should be purchased
At least 10 percent of the total investment should be in alternatives 1 and 2
combined Dividend for the year should be at least $10,000
If the objective is to maximize total dollar return (from both growth and dividends), formulate the LP model for determining the optimal number of shares to purchase of each investment alternative. (Assume that this is a one-year model and that fractional shares of securities may be purchased)

QUESTION FIVE

a) M & K Contractor pays his subcontractors a fixed fee plus mileage or travelling expenses for work per formed. On a given day, the contractor is faced with three electrical jobs associated with various projects (A, B and C). M & K contractor has four electrical subcontractors (Wesside, Federal, National and Universal) who are located at various places throughout the area. Given below are the distances in kilometres) between the subcontractors and the projects.

		Projects		
		A	**B**	**C**
		km	km	km
	Wesside	50	36	16
Subcontractors	Federal	28	30	18
	Naitonal	35	32	20
	Universal	25	25	14

Required:

i) Represent the above problem in a network.
ii) Determine how subcontractors should be assigned in order to minimise the total costs.

b) Explain the assumptions of the quantitative techniques you have used to solve part (a) above.

(Q6 Dec. 1999)

QUESTION SIX

Regal Investments has just received instructions from a client to invest in two shares; one an airline share, the other an insurance share. The total maximum appreciation in share value over the next year is to be maximized subject to the following restrictions:

- the total investment shall not exceed Sh.100,000
- at most Sh.40,000 is to be invested in the insurance shares
- quarterly dividends must total at least Sh.2,600

The airline share is currently selling for Sh.40 per share and its quarterly dividend is Sh.1per share. The insurance share is currently selling for Sh.50 per share and the quarterly dividend is Sh.1.50 per share. Regal's analysts predict that over the next year, the value of the airline share will increase by Sh.2 per share and the value of the insurance share will increase by Sh.3 per share.
A computer software provided the following part solution output:

Objective Function Value = 5,400

Variable	Number	Reduced cost
Airline shares	1,500	0.000
Insurance shares	800	0.000

Constraint	Slack/Surplus	Dual prices
Total investment	0.000	0.050
Investment in insurance	0.000	0.010
Dividends	100.000	0.000

Objective Coefficient Ranges

Variable	Lower limit	Current value	Upper limit
Airline share	2.500	3.000	No upper limit
Insurance share	0.000	2.000	2.400

Right-hand Side Ranges

Constraint	Lower limit	Current value	Upper limit
Total investment	96,000.00	100,000	No upper limit
Investment in insurance	20,000.00	40,000	100,000.00
Dividends	No lower limit	2,600	2,700.00

Required:

a) Formulate the above problem.
b) Explain what reduced cost and dual prices columns above mean.
c) How should the client's money be invested to satisfy the restrictions?
d) Suppose Regal's estimate of the airline shares appreciation is an error, within what limits must the actual appreciation lie for the answer in (c) above to remain optimal?

(Q 6 Dec 2001)

QUESTION SEVEN

a) A baker makes two products; large loaves and small round loaves. He can sell up to 280 of the large loaves and up to 400 small round loaves per day. Each large loaf occupies $0.01m^3$ of shelf space, each small loaf occupies $0.008m^3$ of space, and there is $4m^3$ of shelf space available. There are 8 hours available each night for baking, and he can produce large loaves at the rate of 40 per hour, and small loaves at the rate of 80 per hour. The profit on each large loaf is Sh.5.00 and Sh.3.00 profit on the small round loaf.

Required:
In order to maximize profits, how many large and small round loaves should he produce?

b) Summarize the procedure for solving the kind of quantitative technique you have used to solve part (a) above.

(Q 6 June 2001)

QUESTION EIGHT

a) A small company will be introducing a new line of lightweight bicycle frames to be made from special aluminium alloy and steel alloy. The frames will be produced in two models, deluxe and professional. The anticipated unit profits are currently Sh.1,000 for a deluxe frame and Sh.1,500 for a professional frame. The number of kilogrammes of each alloy needed per frame is summarized in the table below. A supplier delivers 100 kilogrammes of the aluminium alloy and 80 kilogrammes of the steel alloy weekly.

	Aluminium alloy	Steel alloy
Deluxe	2	3
Professional	4	2

Required:

i) Determine the optimal weekly production schedule.

ii) Within what limits must the unit profits lie for each of the frames for this solution to remain optimal?

b) Explain the limitations of the technique you have used to solve part (a) above.

(Q 6 Dec 2000)

QUESTION NINE

a) Define the following terms as used in linear programming:

i) Feasible solution

ii) Transportation problem

iii) Assignment problem

b) The TamuTamu products company ltd is considering an expansion into five new sales districts. The company has been able to hire four new experienced salespersons. Upon analysing the new salesperson's past experience in combination with a personality test which was given to them, the company assigned a rating to each of the salespersons for each of the districts .These ratings are as follows:

		Districts				
		1	2	3	4	5
	A	92	90	94	91	83
Salespersons	B	84	88	96	82	81
	C	90	90	93	86	93
	D	78	94	89	84	88

The company knows that with four salespersons, only four of the five potential districts can be covered.

Required:

i) The four districts that the salespersons should be assigned to in order to maximize the total of the ratings

ii) Maximum total rating.

(Q 6 June 2002)

QUESTION TEN

a) Explain the value of sensitivity analysis in linear programming problems and show how dual values are useful in identifying the price worth paying to relax constraints.

b) J.A Computers is a small manufacturer of personal computers. It concentrates on production of three models- a Desktop 386, a Desktop 286, and a Laptop 486, each containing one CPU Chip. Due to its limited assembly facilities JA Computers are unable to produce more than 500 desktop models or more than 250 Laptop models per month. It has one hundred and twenty 80386 chips (these are used in Desktop-386) and four hundred 80286 chips (used in desktop 286 and Laptop 486) for the month. The

Desktop 386 model requires five hours of production time, the Desktop 286 model requires four hours of production time, and the Laptop 486 requires three hours of production time. J.A Computers have 2000 hours of production time available for the coming month. The company estimates that the profit on Desktop 386 is Sh. 5,000. for a desktop 286 the profit is Sh.3,400 and Sh.3,000 profit for a laptop 486.

Required:
Formulate this problem as a profit maximization problem and mention the basic assumptions that are inherent in such models.

c) An extract of the output from a computer package for this problem is given below: Output solution
$X_1=120$, $X_2=200$, $X_3=200$
Dual values Constraints 3 150
Constraints 4 90
Constraints 5 20

Sensitivity analysis of objective function coefficients:

Variable	Lower limit	Original value	Upper limit
X_1	100	250	No limit
X_2	150	170	200
X_3	127.5	150	170

Sensitivity analysis on R.H.S ranges.

Constraints	Lower limit	Original value	Upper limit
1	320	500	No limit
2	200	250	No limit
3	80	120	130
4	350	400	412.5
5	1950	2000	2180

X_1=Monthly production level for Desktop 386.
X_2 =Monthly production level for Desktop 286.
X_3=Monthly production level for Laptop 486.

Required:
i) Interpret the output clearly, including optimum product mix, monthly profit, unused resources and dual values
ii) Explain the purpose of upper limits and lower limits for the variables X_1,X_2,X_3 and constraints 1 to 5.
iii) Calculate the increase in profit if the company is able to produce a further 10 CPU 80386 chips.

(Q 7 July 2000 Pilot paper)

QUESTION ELEVEN

a) Explain the following terms as used in the context of linear programming:
i) Linear program
ii) Basic feasible solution
iii) Degeneracy
iv) Alternative optimal solutions
v) Redundant constraints
vi) Range of optimality
vii) Artificial variables
viii) Reduced cost
ix) Dummy destination
x) Assignment problem

b) Kenya Canning Transport is to move goods from three factories situated in thika, Naivasha and Machakos. These goods will be distributed to Nairobi, Mombasa and Kisumu. Informaion about the units (in kilograms) to be moved is given below:

Factory	Supply (Kg)	Demand (Kg)	Destination
Thika	200	250	Nairobi
Naivasha	100	125	Mombasa
Machakos	150	125	Kisumu

Naivasha cannot transport to Mombasa.
The costs per Kg. (in shillings) are as follows:

	Nairobi	Mombasa	Kisumu
	Sh.	Sh.	Sh.
Thika	2	8	5
Naivasha	6	12	9
Machakos	4	5	10

Required:
i) Draw a network diagram for the above problem
ii) Set a linear programming model for this problem

NOTE: Do not solve. (Q 5 Dec 1995)

QUESTION TWELVE

a) Explain and graphically show the following terms as used in linear programming:
i) Alternate optimal or multiple solutions
ii) Unbound solutions
iii) Infeasible solutions

b) The Copper Wire Company is a manufacturer of copper wires. Ben Muturi, the company executive director has just received a memo from Stella Musingo, one of his young engineers. Stella informs Ben that; with the forthcoming purchase of new blast furnace to produce copper, it seems an opportune time to completely review their method of producing copper. Currently, a variety of ingredients are entered into the blast furnace in order to produce copper. The production manager, Joe Kiilu makes the decision, as he has for the last 25 years, on the variety of ingredients. It is not an easy decision. The company wants to keep the costs of producing copper down
The ingredients into the blast furnace have to interact to produce copper. Furthermore, the chemical reactions have to be such that the copper produced is viscous enough to flow out of the furnace.
Stella has identified the problems with the current method. First, Joe is retiring in six months and there is no obvious replacement to him. Second, the company never knows if they are minimizing the production costs. Third, sometimes the furnace does not freeze-up. Stella believes that there is a quantitative technique, which can be used to decide what ingredients to enter in to the furnace. Once the model has been developed, special skills such as Joe's will no longer be required. The company will also be confident that they are minimizing costs. In addition, they would be able to eliminate the furnace freeze-up problem

Required:
i) What potential quantitative technique can be used to solve this problem? Why?
ii) Specify the general formulation of the quantitative technique. (Be sure to clearly spell out the meaning of all the variables or parameters used)
iii) If a group were formed to develop an appropriate model, what organizational personnel should be included? What specific skills would each provide?

(Q 6 June 1998)

QUESTION THIRTEEN

a) Briefly and clearly compare and contrast between a transportation problem and an assignment problem as used in linear programming.

b) Downtown Motors Company (DMC), has hired a market services firm to develop an advertising strategy for promoting Downtown's used car sales. The marketing firm has recommended that Downtown use spot announcements on both television and radio as the advertising media for the proposed promotional campaign. Advertising strategy guidelines are expressed as follows:

1. Use at least 30 announcements for combined television and radio coverage.
2. Do not use more than 25 radio announcements.
3. The number of radio announcements cannot be less than the number of television announcements. The television station has quoted a cost of Sh.1,200 per spot announcement and the radio station has quoted a cost of Sh.300 per spot announcement. Downtown's advertising budget has been set at Sh.25,500. The marketing services firm has rated the various advertising media in terms of audience coverage and recall power of the advertisement. For Downtown's media alternatives, the television announcement is rated at 600 and the radio announcement is rated at 200.

Required:

Perform an analysis of advertising strategy for DMC. Include a consideration of the following items:

i) Determine the optimal number of television and radio spot announcements.
ii) Explain the relative merits of each advertising medium.
iii) Determine the rating that would be necessary in order to increase the number of television spots.
iv) Which restrictions placed on the advertising strategy would DMC want to consider relaxing or altering? Why?
v) What is the use of any possible increase in the advertising budget?

(Q6 Dec 1998)

DECISION THEORY

QUESTION ONE
The following is a payoff table for a particular venture.

		States of nature				
		θ_1	θ_2	θ_3	θ_4	θ_5
	D_1	150	225	180	210	250
Decision	D_2	180	140	200	160	225
Alternatives	D_3	220	185	195	190	180
	D_4	190	210	230	200	160

Determine the optimal decision using:
a) Max-min criterion.
b) Max-max criterion.
c) Min-max regret criterion.
d) Maximum expected payoff (assuming equal likelihood of states of nature).

QUESTION TWO
Assume that Table question 1, is a loss table rather than a payoff table. Determine the optimal decision using:
a) The min-max criterion,
b) The min-min criterion,
c) The min-max regret criterion, and
d) The minimum expected loss criterion (again assuming equal likelihood of states of nature).

QUESTION THREE
The following table is a payoff table for a particular venture.

		States of nature					
		θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
	D_1	280	300	260	360	400	450
Decision	D_2	320	420	540	300	280	380
Alternative	D_3	200	360	400	440	250	320
	D_4	350	260	390	500	380	260

The relative likelihood's of occurrence for the states of nature are f $(\theta_1) = 0.18$, f $(\theta_2) = 0.10$, f $(\theta_3) = 0.16$, f $(\theta_4) = 0.24$, f $(\theta_5) = 0.20$, and f $(\theta_6) = 0.12$.

Required:
a) Determine the decision alternative that maximizes expected payoff.
b) Determine expected value under certainty.
c) What is the expected value of perfect information?

QUESTION FOUR
A trust officer for a major banking institution is planning the investment of a $ 1 million family trust for the coming year. The trust officer has identified a portfolio of stocks and another group of bonds that might be selected for investment. The family trust can be invested in stocks or bonds exclusively, or a mix of the two. This trust officer prefers to divide the funds in increments of 10 percent; that is, the family trust may be split 100 percent stocks /0 percent bonds, 90 percent stocks /10 percent bonds, 80 percent stocks /20 percent bonds, and so on.
The trust officer has evaluated the relationship between the yields on the different investments and general economic conditions. Her judgment is as follows:
1) If the next year is characterized by solid growth in the economy, bonds will yield 12 percent and stocks 20 percent.
2) If the next year is characterized by inflation, bonds will yield 18 percent and stocks 10 percent.
3) If the next year is characterized by stagnation, bonds will yield 12 percent and stocks 8 percent.

a) Formulate a payoff table where payoffs represent the annual yield, in dollars, associated with the different investment strategies and the occurrence of various economic conditions
b) Determine the optimal investment strategy using the max-max, max-min, Hurwicz ($\propto = 0.4$), equally likely, and regret criteria.
c) Suppose that a leading economic forecasting firm projects P(solid growth)= 0.4, P(inflation)=0.25, and P (stagnation)= 0.35. Use the expected value criterion to select the appropriate strategy.
d) What is the expected value with perfect information?

QUESTION FIVE

An urban cable television company is investigating the installation of cable TV system in urban areas. The engineering department estimates the cost of the system (in present worth Sh.) to be Sh. 7 million. The sales department has investigated four pricing plans. For each pricing plan, the marketing division has estimated the revenue per household in present worth Sh. to be:

Plan	Revenue per household (Sh.)
I	150
II	180
III	200
IV	240

The sales department estimates that the number of household subscribers would be approximately, either 10,000, 20,000, 30,000, 40,000, 50,000 or 60,000.

Required:

a) Construct a payoff table for this problem.
b) What would be the company's optimal decision under the optimistic approach and the minimax regret approach.
c) Suppose that the sales department has determined the number of subscribers will be a function of the pricing plan.
The probability distributions for the pricing plans are given below.

	Probability under pricing plan.			
Number of subscribers	I	II	III	IV
10,000	0	0.05	0.10	0.20
20,000	0.05	0.10	0.20	0.25
30,000	0.05	0.20	0.20	0.25
40,000	0.40	0.30	0.20	0.15
50,000	0.30	0.20	0.20	0.10
60,000	0.20	0.15	0.10	0.05

Which pricing plan is optimal?

d) Briefly explain the main difference between the approaches used in part (b) and (c) above.

(Q 7 Dec 2001)

QUESTION SIX

Explain the following terms as used in decision analysis.

a) Decision making under risk versus uncertainty.
b) Decision trees versus probability trees.
c) Minimax versus maximax criterion.
d) Pure strategy versus mixed strategy games.
e) Games with more than two persons versus non zero-sum games

(Q 7 Dec 2000)

QUESTION SEVEN

a) Define the following terms used in game theory:
   i) Dominance.
   ii) Saddle point
   iii) Mixed strategy
   iv) Value of the game

b) Consider the two person zero sum game between players A and B given by the following pay-off table:

		Player B Strategies			
		A	B	C	D
Player A Strategies	1	2	2	3	-1
	2	4	3	2	6

Required:

i) Using the maximin and minimax values, is it possible to determine the value of the game? Give reasons.

ii) Use graphical methods to determine optimal mixed strategy for player A and determine the value of the game.

(Q7 June 2000)

NETWORK ANALYSIS

QUESTION ONE

Consider the simplified scenario for the development of a consumer product through the market test phase shown in the table below.

Activity	Symbol	Preceding activities	Time estimated (weeks)
Design promotion campaign	A	-	3
Initial pricing analysis	B	-	1
Product design	C	-	5
Promotional costs analysis	D	A	1
Manufacture prototype models	E	C	6
Product cost analysis	F	E	1
Final pricing analysis	G	B,D,F	2
Market test	H	G	8

a) Draw the network for this project.
b) Calculate the slacks and interpret their meaning.
c) Determine the critical path and interpret its meaning.
d) Construct a time chart and identify scheduling flexibilities.

QUESTION TWO

a) For the product development project in question 1 consider the detailed time estimates given in the following table. Note that time estimates in the preceding exercise are equivalent to modal time estimates in this exercise.

Time Estimates (weeks)

Activity	Optimistic	Most likely	Pessimistic
A	1	3	4
B	1	1	2
C	4	5	9
D	1	1	1
E	4	6	12
F	1	1	2
G	1	2	3
H	6	8	10

Re-label your network in the question 1 to include expected duration $\bar{d}_{ij}$ (in place of activity duration d_{ij} and variances σ_{ij}.

Use equations below

$$\bar{d}_{ij} = \frac{a_{ij} + 4m_{ij} + b_{ij}}{6} \quad \text{and} \quad \sigma_{ij}^2 = \left(\frac{b_{ij} - a_{ij}}{6}\right)^2$$

$$6\sqrt{\sigma_{ij}} = b_{ij} - a_{ij} \quad \text{or}$$

where:
a_{ij} - optimistic time
b_{ij} - pessimistic time
m_{ij} - most likely time

b) Compare slacks to those in question 1.
c) Has the critical path changed?
d) Determine the following probabilities:
   i) That the project will be completed in 22 weeks or less.
   ii) That the project will be completed by its earliest expected completion date.
   iii) That the project takes more than 30 weeks to complete.

QUESTION THREE
Consider the cost-time estimates for the product development project of question one above as given in the table below.

	Time Estimates (weeks)		Direct Cost Estimates ($ '000')	
Activity	Normal	Crash	Normal	Crash
A	3	1.0	3.5	10.0
B	1	0.5	1.2	2.0
C	5	3.0	9.0	18.0
D	1	0.7	1.0	2.0
E	6	3.0	20.0	50.0
F	1	0.5	2.2	3.0
G	2	1.0	4.0	9.0
H	8	6.0	100.0	150.0

Indirect cost is made up of two components: Fixed cost of $5,000 and a variable cost of $1,000 per week of elapsed time. Also, for each week the project exceeds 17 weeks, an opportunity cost of $2,000 per week is assessed.

Construct a time chart for the minimum total cost schedule.

QUESTION FOUR
Consider a project which has been modelled as follows:

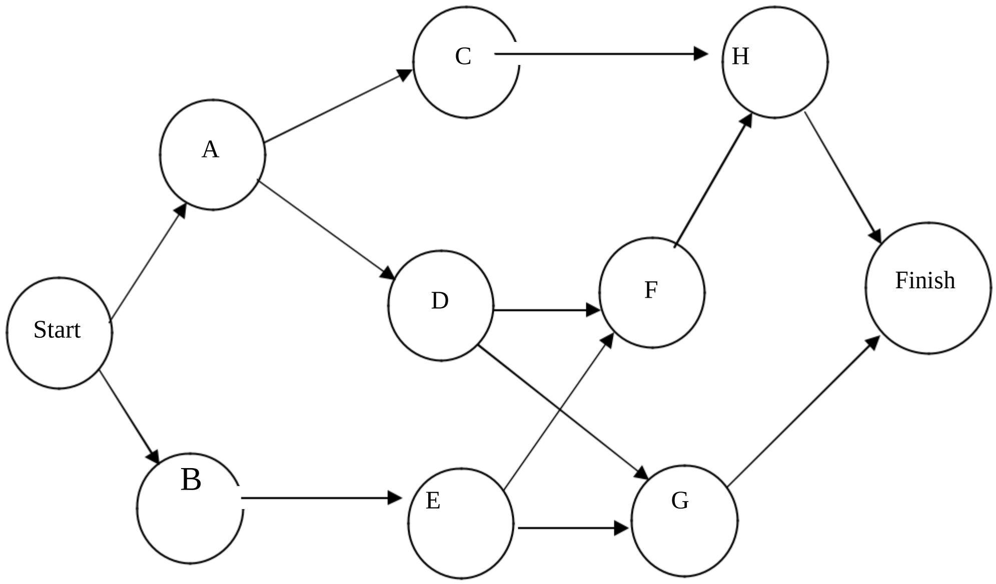
Activity	Immediate Predecessor (s)	Completion Time (hours)
A	-	7
B	-	10
C	A	4
D	A	30
E	A	7
F	B, C	12
G	B, C	15
H	E, F	11
I	E, F	25
J	E, F	6
K	D, H	21
L	G, J	25

Required:
a) Determine the project's expected completion time and its critical path.
b) Can activities E and G be performed at the same time without delaying the completion of the project?
c) Can one person perform activitiesA, G and I without delaying the project?
d) By how much time can activities G and L be delayed without delaying the entire project?
e) By how much time would the project be delayed if activity G were delayed by 3 hours and activity L by 4 hours? Explain.

(Q 8 Dec 2001)

QUESTION FIVE
a) Explain the following terms used in network analysis:
   i) Network planning;
   ii) Activities;
   iii) Events;
   iv) Critical path;
   v) Float.

b) Consider the following project network and activity times (in weeks).



Activity	Activity Times (weeks)
A	5
B	3
C	7
D	6
E	7
F	3
G	10
H	8

Required:

i) How long will it take to finish this project?
ii) Can activity D be delayed without delaying the entire project? If so, by how many weeks?
iii) What is the schedule for activity E?

(Q 8 June 2001)

QUESTION SIX

a) A small construction project involves the following activities:

	Activity		Normal Time (days)	Normal Cost (Sh.)	Crash Time (days)	Crash Cost (Sh.)
1.2	Clear ground	A	6	60,000	5	70,000
1.3	Lay foundation	B	5	30,000	3	50,000
2.4	Build walls	C	3	10,000	2	15,000
3.4	Roofing and piping	D	7	40,000	4	55,000
3.5	Painting	E	4	20,000	3	30,000
4.5	Landscaping	F	2	10,000	1	17,500

Required:

i) Determine the shortest time and associated cost to finish this project.
ii) If a penalty of Sh.4,500 must be charged for every day beyond 12 days, what is the most economical time for completing the project?

b) Explain the four different methods or approaches for organizing and displaying project information.

(Q 8 Dec 2000)

QUESTION SEVEN

a) State four attributes of the Beta distribution which made it to be chosen as representing the distribution of activity times in project evaluation and review technique (PERT).

b) Lillian Wambugu is the project manager of Jokete Construction Company. The company is bidding a contract to install telephone lines in a small town. It has identified the following activities along with their predecessor restrictions, expected times and worker requirements:

Activity	Predecessors	Duration Weeks	Crew Size Workers
A	-	4	4
B	-	7	2
C	A	3	2
D	A	3	4
E	B	2	6
F	B	2	3
G	D, E	2	3
H	F, G	3	4

Lillian Wambugu has agreed with the client that the project should be completed in the shortest duration.

Required:

i) Draw a network for the project.
ii) Determine the critical path and the shortest project duration.
iii) Lillian Wambugu will assign a fixed number of workers to the project for its entire duration and so she would like to ensure that the minimum number of workers is assigned and that the project will be completed in 14 weeks.

Draw a schedule showing how the project will be completed in 14 weeks

iv) Comment on the schedule you have drawn in (iii) above.

(Q 8 June 2002)

ANSWERS - PAST PAPERS

TOPIC 1

QUESTION 1

i) When $p_s = 5$, P is as follows:

$P_5 = 50000 \times 5 - 6250 \times 5^2$
$= Sh.93750$

When $p_s = 6$, P is

$P_6 = 50000 \times 6 - 6250 \times 6^2$
$= Sh.75000$

When $p_s = 4$, P is

$P_4 = 50000 \times 4 - 6250 \times 4^2$
$= Sh.100000$

Since $P_4 > P_5 > P_6$ where P_p is Profit at a given price p, then the function is decreasing at $p_s = 5$

ii) When $p_s = 4.5$, $P = 50000 \times 4.5 - 6250 \times 4.5^2$
$= 98437.5$

The profit will increase by 98437.5 – 93750 = Sh.4687.50

iii) The first differentiation of profit with respect to price equated to zero is as follows:

$$\frac{dP}{dp_s} = 50000 - 12500p = 0 \Rightarrow p_s = \frac{50000}{12500} = 4$$

The second differentiation with respect to price gives

$$\frac{d^2P}{dp_s^2} = -12500$$

Since $\frac{d^2P}{dp_s^2} < 0$ it means it is a maximization function.

QUESTION 2

Let profit = z

Profit z = PQ – C
$= (12 - 0.4Q)Q - (5 + 4Q + 0.6Q^2)$
$= 12Q - 0.4Q^2 - 5 - 4Q - 0.6Q^2$
$= 8Q - Q^2 - 5$

For maximum profit, the differentiation of z with respect to Q equals zero.

$$\frac{dz}{dQ} = 8 - 2Q = 0 \Rightarrow 2Q = 8 \Rightarrow Q = 4$$

So P = 12 – 0.4Q and for Q = 4
$= 12 - 1.6$
$= 10.4$

$\frac{d^2z}{dQ^2} = -2Q < 0$ Profit is maximized.

Profit is maximised at a price of 10.4 and when quantity = 4

To maximize sales then,

$$\frac{d(PQ)}{dQ} = \frac{d(12Q - 0.4Q^2)}{dQ} = 0$$

$=12–0.8Q=0$

$\Rightarrow Q = \frac{12}{0.8} = 15$ and since $\frac{d^2(PQ)}{dQ^2} = -0.8 < 0$ then sales is maximized

So $P = 12 – 0.4 \times 15$

$= 6$

QUESTION 3

a) Taking the following to mean: TC – Total cost

AC – Average cost

MC – Marginal cost

Q – Number of units

Then $AC = \frac{TC}{Q}$

And $MC = \frac{d(TC)}{dQ}$

These are the relationships that link TC, AC, and MC.

To comment on the CPA students analysis,

The derivative of AC is as follows,

$$\frac{d(AC)}{dQ} = \frac{d(TC/Q)}{dQ} = \frac{\frac{d(TC)}{dQ} \times Q - TC}{Q^2} = \frac{1}{Q}\frac{d(TC)}{dQ} - \frac{TC}{Q^2}$$

Since $\frac{d(AC)}{dQ} \neq \frac{d(TC)}{dQ} = MC$ then the students comment is wrong in getting marginal cost. The student is right though in saying that $AC = \frac{TC}{Q}$.

b)

i) Total cost function can be obtained from expression of marginal cost (MC)

since, $\frac{d(TC)}{dQ} = (MC)$ Then

$dTC = (MC)dQ$

Integrating both sides gives:

$TC = \int (MC)dQ$

Given $MC = Q^2 – 28Q + 211$

then $TC = \int (Q^2 - 28Q + 211)dQ$

$= \frac{Q^3}{3} - \frac{28Q^2}{2} + 211Q + A$

A – is a constant of integration.

Given that when Q = 0,TC = Sh 10 million

then A =10

So the total cost function is as follows:

$$TC = \frac{Q^3}{3} - 14Q^2 + 211Q + 10$$

ii) The total revenue (TR) function can be obtained from Average revenue (AR) function as follows,

$$AR = \frac{TR}{Q} \quad \text{So} \quad TR = Q \times AR$$

$$= Q \times (200 - 8Q)$$

$$= 200Q - 8Q^2$$

iii) Profit equal to TR – TC. Since TR and TC expression have been obtained from (i) and (ii), then profit P is as follows,

$$P = TR - TC$$

$$= 200Q - 8Q^2 - \left(\frac{Q^3}{3} - 14Q^2 + 211Q + 10\right)$$

$$= 11Q + 6Q^2 - \frac{Q^3}{3} - 10$$

iv) The level of output that maximizes profit is got by equating the derivative of profit P with respect to Q to zero, as follows

$$\frac{d(P)}{dQ} = \frac{d(11Q + 6Q^2 - \frac{Q^3}{3} - 10)}{dQ} = 0$$

$$= 11 + 12Q - Q^2 = 0$$

The solution to this quadratic equation is as follows:

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where a, b, c are the coefficients of the equation as follows:
a = - 1, b = 12, and c = 11.

$$\text{So} \quad Q = \frac{-12 \pm \sqrt{12^2 - 4 \times (-1) \times 11}}{2 \times (-1)}$$

$$\text{So } Q = \frac{-12 + 10}{-2} = 1 \text{ or } Q = \frac{-12 - 1}{-2} = 11$$

Since two points of maximum profit exist, then the Q that gives more profit is the one to be used. At Q = 11,

$$P = 11 \times 11 + 6 \times 11^2 - \frac{11^3}{3} - 10$$

= 151.333 million

At Q = 1,

$$P = -11 \times 1 + 6 \times 1^2 - \frac{1^3}{3} - 10$$

= 15.333 million

So the level that maximizes profit is Q = 11.

v) Marginal revenue (MR) can be obtained from Total revenue (TR) as follows: $\frac{d(TR)}{dQ} = MR$

TR = 200Q – 8Q2

So $\frac{d(TR)}{dQ} = \frac{d(200Q - 8Q^2)}{dQ}$

= 200 – 16Q

QUESTION 4

i) The initial cost of the project is determined when the time is zero. That is when the project is started. Given Profit P = 10x – x2 – 5, then the initial cost of the project is when x = 0.
Profit = 10 × 0 – 02 – 5 = -
5 The initial cost is sh. 5000.

ii) Equating the profit function to zero and solving the function for the time determines break-even time in months for the project.
P = 10x – x2 – 5 = 0
Since this is a quadratic equation, the solution is as follows,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Given a = - 1
b = 10
c = - 5

Then

$$x = \frac{-10 + \sqrt{10^2 - 4\times(-1)\times(-5)}}{2\times(-1)} \quad \text{or} \quad \frac{-10 - \sqrt{10^2 - 4\times(-1)*(-5)}}{2\times(-1)}$$

$$= \frac{-10 + 8.94}{-2} \quad \text{or} \quad \frac{-10 - 8.94}{-2}$$

= 0.527 or 9.472 months

Break-even time is 0.527 and 9.472 months.

iii) The best time to end the project is when profit is maximum. This is determined by differentiating the profit function with respect to time and equating to zero as follows:

$$\frac{dP}{dx} = 10 - 2x = 0$$

$\Rightarrow$ x = 5

The best time to end the project is after 5 months.

iv) To obtain the total profit within the break-even points, the profit function is integrated within those break-even points as follows,

$$\text{Profit} = \int_{0.527}^{9.472} Pdx \int_{0.527}^{9.472} (10 - x_2 - 5)\, dx$$

$$= \left[\frac{10x^2}{2} - \frac{x^3}{3} - 5x\right]_{0.527}^{9.472}$$

$$= \left(5\times 9.472^2\right) - \frac{(9.472)^3}{3} - (5\times 9.472) - \left(5\times 0.527^2\right) - \frac{(0.527)^3}{3} - (5\times 0.527)$$

= 117.96 – (- 1.30) = 119.26 × 1000
= Sh. 119,260

QUESTION 5

a)

i) To obtain the time when there are a maximum number of people queuing, the derivative of the equation is equated to zero.

$$\frac{dy}{dx} = \frac{d\left(x^3 - 14x^2 + 50x\right)}{dx} = 0$$

$$= 3x^2 - 28x + 50 = 0$$

This is a quadratic equation with the following solutions,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Given a = 3
b = - 28
c = 50

Then

$$x = \frac{28 + \sqrt{28^2 - 4\times(50)\times(3)}}{2\times(3)} \quad \text{or} \quad \frac{28 - \sqrt{28^2 - 4\times(50)\times(3)}}{2\times(3)}$$

$$= \frac{28 + 13.56}{6} \text{ or } \frac{28 - 13.56}{6}$$

= 6.92 or 2.41

The number of shoppers queuing at these particular times is as follows,

$y = 6.92^3 - 14 \times (6.92)^2 + 50 \times 6.92$
$= 6.96$

or $y = 2.41^3 - 14 \times (2.41)^2 + 50 \times 2.41$
$= 53.18$

The management should deploy more cashiers after 2.41 hours, that is at 11.25 am. The number of people queuing at this particular time is 53.

ii) The number of man hours spent is equal to Y × x. To get the man-hours spent per day, the function is integrated within the limits 0 ≤ x ≤ 8.5

$$\int_0^{8.5} (Yx)\partial x = \int_0^{8.5} (x^4 - 14x^3 + 50x^2)dx$$

$x^5 - 14x^4 + 50x^3$ 8.5

543_0

= 839.3 – 0 = 839.3 man-hours

b)

i) Average cost AC is given by the following expression. $AC = \frac{TC}{x}$ where TC – Total cost

$TC = \int (MC)dx$ and given MC = $x^2 - x + 2$

Then $TC = \int (x^2 - x + 2)dx$

$= \frac{x^3}{3} - \frac{x^2}{2} + 2x + A$ where A is a constant of integration.

Given the information that when x = 0; TC = 500, then A is determined as follows,

$500 = \frac{0^3}{2} - \frac{0^2}{} + 2\times 0 + A$ 3

$\Rightarrow A = 500$

So the TC function is as follows,

$\frac{x^3}{3} - \frac{x^2}{2} + 2x + 500$

The AC function then is

$\frac{TC}{x} = \frac{x^2}{3} - \frac{x}{2} + \frac{500}{x}$

Average revenue AR is given by the following expression

$AR = \frac{R}{x}$ Given R = $- x^2 + 5x$ then

AR = - x + 5

ii) Profit maximizing output is obtained by equating the differential of profit to zero and solving for the x values as follows:
Profit P = R – TC

$= -x^2 \quad \frac{x^3}{3} - \frac{x^2}{2} + 2x + 500$

$= -\frac{x^2}{500\ 2} - \frac{x^3}{3} + 3x -$

$\frac{dP}{dx} = 0 = -1x - x^2 + 3$ $\qquad x^2 + x - 3 = 0$

Since this is a quadratic equation, the solutions are obtained as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Given a = - 1
b = - 1
c = 3

Then

$$x = \frac{1+\sqrt{1^2 - 4\times(-1)\times(3)}}{2\times(-1)} \quad \text{or} \quad \frac{1-\sqrt{1^2 - 4\times(-1)\times(3)}}{2\times(-1)}$$

= - 2.3 or 1.3

So x = 1300 calculators.
Price to charge = AR = - 1.3 + 5 = sh. 3700

Cost per calculator = AC = $\frac{1.3^2}{3} - \frac{1.3}{2} + \frac{500}{1.3}$ = Sh. 384.5

QUESTION 6

a)

i) Transition matrix is that which contains the probabilities of moving from any one state to another.
ii) Initial probability vector is the vector that contains the current state before transition.
iii) Equilibrium state is the state that a system settles on in the long run.
iv) Absorbing state is one in which cannot be left once entered. It has a transition probability of unity to itself and of zero to other states.

b)

i) To make the transition matrix, we can make a loss probability table and retention probability table first.

Loss Probability table

	To A	To B	To C	To D	Total Loss
From A	0	0	0	0.05	0.05
From B	0.2	0	0	0.1	0.3
From C	0	0.2	0	0.15	0.35
From D	0	0	0.2	0	0.2

Retention Probability table

	Retention=(1 – Total loss)		
A	1 – 0.05	=	0.95
B	1 – 0.3	=	0.7
C	1 – 0.35	=	0.65
D	1 – 0.2	=	0.8

So the transition matrix will be as follows.

		To			
		A	B	C	D
From	A	0.95	0	0	0.05
	B	0.2	0.7	0	0.1
	C	0	0.2	0.65	0.15
	D	0	0	0.2	0.8

NOTE:

1) D only looses to C, since whatever it looses due to separation is immediately replenished. i.e. 25% loss is immediately returned by more employment. So the loss is zero.
2) The retentions are placed on the diagonal of transition matrix before the loss probabilities.
3) Notice that summation in rows of transition matrix is equal to one.
4) The matrix can be interchanged to be

		From			
		A	B	C	D
To	A	0.95	0.2	0	0
	B	0	0.7	0.2	0
	C	0	0	0.65	0.2
	D	0.05	0.1	0.15	0.8

In this case the initial vector is post multiplied as follows⇒Transition matrix× Initial vector

ii) To get the initial probability vector after two periods, just multiply the initial vector with the transition matrix twice.
Initial vector

$$\begin{pmatrix} A & B & C & D \end{pmatrix} = \begin{pmatrix} 60 & 90 & 150 & 200 \end{pmatrix}$$

The first period

$$\begin{pmatrix} 60 & 90 & 150 & 200 \end{pmatrix} \times \begin{matrix} 0.95 & 0 & 0 & 0.05 \\ 0.2 & 0.7 & 0 & 0.1 \\ 0 & 0.2 & 0.65 & 0.15 \\ 0 & 0 & 0.2 & 0.8 \end{matrix} = \begin{pmatrix} 75 & 93 & 137.5 & 194.5 \end{pmatrix}$$

Calculations

- 60×0.95+90×0.2+150×0+200×0=75
- 60×0+90×0.7+150×0.2+200×0=93
- 60×0+90×0+150×0.65+200×0.2=137.5
- 60×0.05+90×0.1+150×0.15+200×0.8=194.5

The second period.

$$\begin{pmatrix} 75 & 93 & 137.5 & 194.5 \end{pmatrix} \times \begin{matrix} 0.95 & 0 & 0 & 0.05 \\ 0.2 & 0.7 & 0 & 0.1 \\ 0 & 0.2 & 0.65 & 0.15 \\ 0 & 0 & 0.2 & 0.8 \end{matrix} = \begin{pmatrix} 89.85 & 92.6 & 128.273 & 189.275 \end{pmatrix}$$

Calculations

- 75×0.95+93×0.2+137.5×0+194.5×0=89.85
- 75×0+93×0.7+137.5×0.2+194.5×0=92.6
- 75×0+93×0+137.5×0.65+194.5×0.2=128.273
- 75×0.05+93×0.1+137.5×0.15+194.5×0.8=189.275

Approximately in A ≈ 89, B ≈ 92, C ≈ 128, D ≈ 189

iii) The equilibrium or steady state is determined from the following matrix and equation as follows:

$$\begin{pmatrix} A & B & C & D \end{pmatrix} \times \begin{pmatrix} 0.95 & 0 & 0 & 0.05 \\ 0.2 & 0.7 & 0 & 0.1 \\ 0 & 0.2 & 0.65 & 0.15 \\ 0 & 0 & 0.2 & 0.8 \end{pmatrix} = \begin{pmatrix} A & B & C & D \end{pmatrix} \qquad (1)$$

and $A + B + C + D = 1$ (2)

From the matrix multiplication (1), the following expressions are determined in terms of A.

$0.95A+0.2B=A \Rightarrow 0.05A=0.2B \quad \Rightarrow B=0.25A$ (3)

$0.7B+0.2C=B \Rightarrow 0.075A=0.2C \Rightarrow C=0.375A$ (4)

$0.65C + 0.2D = C \Rightarrow 0.13125A = 0.2D \Rightarrow D = 0.65625A$ (5)

$0.05A+0.1B+0.15C+0.8D=D$ (6)

From equation (2), (3), (4) and (5),

$A + 0.25A + 0.375A + 0.65625A = 1$

$\Rightarrow 2.28125A = 1$

So $A = 0.4384$

$B = 0.25 \times A = 0.25 \times 0.4384 = 0.1096$

$C = 0.375 \times A = 0.375 \times 0.4384 = 0.1644$

$D = 0.65625 \times A = 0.65625 \times 0.4384 = 0.2877$

The number of employees in each class at equilibrium is obtained as follows:

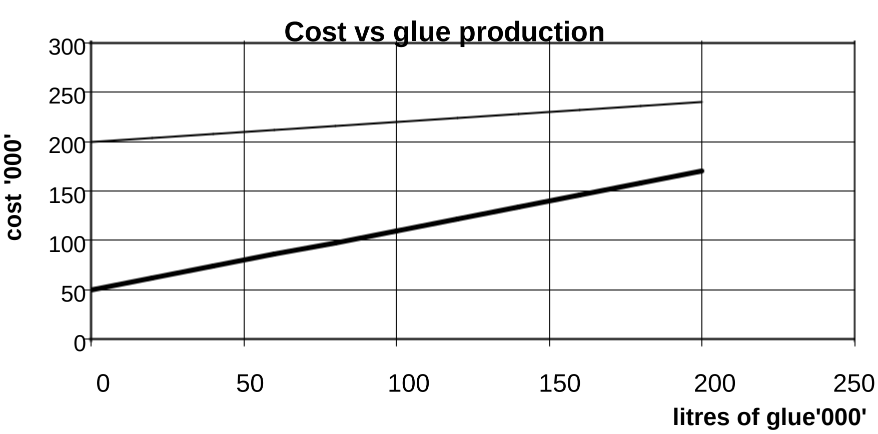
$A = 0.4384 \times 500$, $B = 0.1096 \times 500$, $C = 0.644 \times 500$, $D = 0.2877 \times 500$

$500 = 200 + 150 + 90 + 60$ (the initial state)

So at equilibrium $\begin{pmatrix} A & B & C & D \end{pmatrix} \approx \begin{pmatrix} 219 & 55 & 82 & 144 \end{pmatrix}$

QUESTION 7

a) The graph will look as follows



From the graph, firm B's cost function increases more rapidly than firm A's even though it starts from a lower value of 50,000.

b)

i) Profit=Revenue-costs

Revenue=AR×q=(600-0.5q)×q=600q-0.5q²

Cost is got from the integration of the marginal cost function as follows

$$\text{Cost} = \int MC\,dq = \int \left(140 - 8q + 0.15q^2\right)dq = 140q - \frac{8}{2}q^2 + \frac{0.15}{3}q^3 + A$$

A is a constant of integration and is determined as follows

When q=0, cost=1000. Substituting this in the expression of the cost gives A=1500 So cost=$140q-4q^2+0.05q^3+1500$ and

Profit =$600q-0.5q^2-(140q-4q^2+0.05q^3+1500)$

$=460q+3.5q^2-0.05q^3-1500$

ii) To find at what quantity q_m profit is maximizes, the expression of profit is differentiated and equated to zero as follows:

$$\frac{d(profit)}{dq}=460+7q-0.15q^2=0$$

Solving for the values of q

$$q=\frac{-7\pm\sqrt{7^2-4\times460\times(-0.15)}}{2\times(-0.15)}=\frac{-7\pm18.03}{-0.3}=-36.76 \text{ or } 83.43$$

So q_m=83 biro pens

iii) Revenue is maximized where:

$$\frac{d(revenue)}{dq}=600-q=0\Rightarrow q=600$$ which is different from q_m=83 where profit is maximized

QUESTION 8

a)

i) Turning point is the point of local minima or maxima for a function. This point has zero slope.

ii) Second order derivative condition states that if the first derivative equals zero and the second derivative is defined then the given point is a relative minimum if the second derivative is greater than zero, or maximum if the second derivative is less than zero

iii) Partial derivative is the derivative of a multivariate function (function of more than one variable). It is usually with respect to each of the independent variable.

iv) Mixed or cross partial derivative is obtained by first getting the derivative of multivariate function with respect to one variable then the second derivative with respect to the second variable.

v) A saddle point is a stationery point that is neither a maximum nor a minimum. Here the difference between the product of pure second partial derivative (second derivative of a function with respect to one variable) and square of mixed partial derivative is less than zero

b)

i) Total revenue = Px

= $(100-0.01x)\times x$

= $100x-0.01x^2$

ii) Profit = Revenue-cost

= $100x-0.01x^2-50x-30000$

= $50x-0.01x^2-30000$

$$\frac{d(Profit)}{dx}=50-0.02x=0\Rightarrow x=\frac{50}{0.02}=2500$$ Chicken wings

iii) To maximize revenue

$$\frac{d(Revenue)}{dx}=100-0.02x=0\Rightarrow x=5000$$

So profit when revenue is maximized is

Profit=$50\times5000-0.01\times(5000)^2-30000$=-30,000

Maximum profit =$50\times2500-0.01\times(2500)^2-30000$

125000-0.01×6250000-30000=32,500

So the difference in profit is 32,500-(-30000)=62,500

QUESTION 9

a) The Leontief open model is

$Mx+d=x$

So rearranging the equation $(I-M)x=d$

$x=(I-M)^{-1}d$

Where M-matrix of technical coefficients

x-required production

d-the external demand

I-Identity matrix

Given that $M=\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.1 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.4 \end{pmatrix}$

$d=\begin{pmatrix} 40 \\ 50 \\ 60 \end{pmatrix}$ Then

$$(I-M)=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.1 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.4 \end{pmatrix} = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.2 \\ 0.2 & 0.3 & 0.6 \end{pmatrix}$$

$$(I-M)^{-1} = \frac{1}{\text{Determinant}(I-M)} \times \text{Adjoint}(I-M)$$

Determinant (I-M) = $|I-M| = 0.7(0.36-0.04) - 0.2(0.06-0.04) + 0.1(0.03-0.12)$

$= 0.21 - 0.004 - 0.009 = 0.197$

Ad joint (I-M)=Transpose of the co-factors of (I-M)

$$\text{Co-factors of } (I-M) = \begin{pmatrix} 0.3 & -0.02 & -0.09 \\ -0.09 & 0.4 & -0.17 \\ -0.02 & -0.13 & 0.4 \end{pmatrix}$$

$$\text{So adjoint } (I-M) = \begin{pmatrix} 0.3 & -0.09 & -0.02 \\ -0.02 & 0.4 & -0.13 \\ -0.09 & -0.17 & 0.4 \end{pmatrix}$$

$$\text{And } (I-M)^{-1} = \frac{1}{0.197} \times \begin{pmatrix} 0.3 & -0.09 & -0.02 \\ -0.02 & 0.4 & -0.13 \\ -0.09 & -0.17 & 0.4 \end{pmatrix}$$

$$X = (I-M)^{-1}d = \frac{1}{0.197} \times \begin{pmatrix} 0.3 & -0.09 & -0.02 \\ -0.02 & 0.4 & -0.13 \\ -0.09 & -0.17 & 0.4 \end{pmatrix} \times \begin{pmatrix} 40 \\ 50 \\ 60 \end{pmatrix}$$

$$= \frac{1}{0.197} \times \begin{pmatrix} 0.3\times 40 - 0.09\times 50 - 0.02\times 60 \\ -0.02\times 40 + 0.4\times 50 - 0.13\times 60 \\ -0.09\times 40 - 0.17\times 50 + 0.4\times 60 \end{pmatrix} = \frac{1}{0.197} \times \begin{pmatrix} 6.3 \\ 11.4 \\ 11.9 \end{pmatrix} = \begin{pmatrix} 32.0 \\ 57.9 \\ 60.4 \end{pmatrix}$$

b) If $d = \begin{bmatrix} 30 \\ 55 \\ 70 \end{bmatrix}$ the production vector will be

$$X = (I - M)^{-1} d = \frac{1}{0.197} \times \begin{bmatrix} 0.3 & -0.09 \\ -0.02 & 0.4 \\ -0.09 & -0.17 \end{bmatrix}$$

$$= \frac{1}{0.197} \times \begin{bmatrix} 2.65 \\ 12.35 \\ 15.95 \end{bmatrix} = \begin{bmatrix} 13.45 \\ 62.44 \\ 80.96 \end{bmatrix}$$

TOPIC 2

QUESTION 1

a)

i)

Sector	Weight		July 2001 Index (1994 = 100)		
	W	Summation	P	WP	Summation
Mining and quarrying	41	41	361	14801	14801
Manufacturing:					
Food, drink and tobacco	77		106	8162	
Chemicals	66		109	7194	
Metal	47		72	3384	
Engineering	298		86	25628	
Textiles	67		70	4690	
Other manufacturing	142		91	12922	
Total of manufacturing	**697**	697		**61980**	61980
Construction	182	182	84	15288	15288
Gas, electricity and water	80	80	115	9200	9200
Totals		1000			101269

Index of industrial production is $\frac{\sum WP}{\sum (W)} = \frac{101269}{1000} = 101.269 \approx 101.3$

Index of manufacturing industries is $\frac{\sum (WP)}{\sum (W)} = \frac{61980}{697} = 88.924 \approx 88.9$

ii) The index of manufacturing industries is lower than when it is combines with the other industries. Index of industrial production shows that there was a slight increase of 1.1% from the base year 1994 while manufacturing industries shows that there was a decrease of 11.1% from the base year 1994

b) Uses of index numbers includes the following

- Determination of cost of living
- Determination of income variation with years
- Changes in stock prices
- Relationship between industries and time
- Countries' economies can be compared easily
- Acts as a control measure

c) Limitation of index numbers include:

- There is usually a problem of choice of sample. This means it may not show a good representative
- The exact effect is usually clouded by the general index
- Choice of the best index to represent a given data is usually complicated

QUESTION 2

i) The following table will assist in determination of mean, standard deviation, mode, median.

Income per month	mid-point x/1000	Percentage f	Cumulative frequency	fx	$f(x-\bar{x})^2$
19999.5 - 29999.5	24.9995	5	5	125.00	13462.86
29999.5 - 34999.5	32.4995	2	7	65.00	3940.94
34999.5 - 35999.5	35.4995	3	10	106.50	5139.40
35999.5 - 39999.5	37.9995	0	10	0.00	0.00
39999.5 - 44999.5	42.4995	5	15	212.50	5913.36
44999.5 - 49999.5	47.4995	10	25	475.00	8637.72
49999.5 - 59999.5	54.9995	15	40	824.99	7187.58
59999.5 - 69999.5	64.9995	18	58	1169.99	2544.70
69999.5 - 99999.5	84.9995	21	79	1784.99	1381.21
99999.5 - 149999.5	124.9995	17	96	2124.99	39347.73
149999.5 - 249999.5	199.9995	4	100	800.00	60624.29
		100		7688.95	148179.79

Mean $\bar{x} = 1000\times\frac{\sum fx}{\sum f} = 1000\times\frac{7688.95}{100}$ = Sh.76,889.5

Median determination

Median Frequency: Q_2 frequency= $\frac{1}{2}(n+1) = \frac{1}{2}(100+1) = 50.5$ So Q_2 class interval is 59999.5-69999.5 with its frequency being 18. This is read from cumulative frequency column. The interval and frequency can then be read.

Median= $l_{Q2} + \frac{f_{Q2} - f_{Q1}}{f} \times C_{Q2} = 59999.5 + \frac{50.5-40}{21}\times 10000 = 64{,}951.88$ Shillings

Where
- l_{Q2} - lower boundary of Q_2 class
- f_{Q2} - Q_2 frequency
- f_{Q1} - Frequency of class after Q_2 class
- f - Cumulative frequency at the start of Q_2 class
- C_{Q2} - class interval of the Q_2 class

Mode determination

The highest frequency is 21 so modal class is 69999.5-99999.5

Mode= $l_1 + \frac{f_0 - f_1}{(f_0 - f_1) + (f_0 - f_2)} \times C = 69999.5 + \frac{21-18}{(21-18)+(21-17)} \times 30000 = 82856.64$ Shillings

Where
- l_1 - lower boundary of modal class
- f_0 - Frequency of modal class
- f_1 - Frequency of class just before modal class
- f_2 - Frequency of class just after modal class
- C - Interval of the modal class

Standard deviation $\sigma = 1000\times\sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = 1000\times\sqrt{\frac{148179.79}{100}} = 38494.13$ Shillings

The mean, median and mode show that the mortgages are mostly taken by high income earners. The standard deviation shows that there is a wide variation in income in relation to people taking mortgages.

ii) Descriptive statistics deals with the way raw data is transformed into useful information. For example debt collection from debtors can be studied for a given year. The age debtors report will indicate the number of debtors for a given time. This can be used to approximate the provision for doubtful debts.

Note:

- This is a typical real life problem where there are unequal intervals and open ended frequency distribution
- Notice the way to create **continuity** by including the last digit of 0.5
- The open ended frequencies at the start and end of the distribution is covered up by having two times the class interval just ahead or just preceding

QUESTION 3

a) The data ensuring continuity and to assist in calculation of the mean is as follows

(Diameter-0.97) × 10000	x	No. of component f	fx	$f(x-\bar{x})^2$
46.5 - 49.5	48	2	96	522.5
49.5 - 52.5	51	6	306	1039.7
52.5 - 55.5	54	8	432	826.5
55.5 - 58.5	57	15	855	769.8
58.5 - 61.5	60	42	2520	728.2
61.5 - 64.5	63	68	4284	92.1
64.5 - 67.5	66	49	3234	165.2
67.5 - 70.5	69	25	1725	584.7
70.5 - 73.5	72	18	1296	1105.3
73.5 - 76.5	75	12	900	1409.0
76.5 - 79.5	78	4	312	765.7
79.5 - 82.5	81	1	81	283.5
		250	16041	8292.3

Mean $\bar{x} = 10^{-4} \times \frac{\sum fx}{\sum f} + 0.97 = 10^{-4} \times \frac{16041}{250} + 0.97 = 0.976416$

Standard deviation $\sigma = 10^{-4} \times \frac{\left(\sum f\left(x-\bar{x}\right)\right)}{\sum f} + 0.97 = 10^{-4} \times \frac{8292}{250} + 0.97 = 0.973317$

H_0: $\mu=0.97642$ customer is getting reasonable value

H_1: $\mu\neq0.97642$ customer is not getting reasonable value

The test statistic is $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{0.976416-0.97642}{0.973317/\sqrt{250}} = 0.000065$

From normal distribution tables z at 5% significance level (two tiled test) is 1.96. Since calculated z is less than $z_{5\%}$ then H_0 is accepted. That means the customer is getting reasonable value when the diameter is indicated to be 0.97642.

b)

i) When relative importance of different items is different. It is usually used where intervals are involved

ii) To estimate the median of a grouped frequency distribution graphically:
- Find cumulative frequency at midpoints of class interval.
- Draw the curve of cumulative frequency as the midpoints ogive.
- Get the middle part of frequency by dividing total frequency by 2.
- Draw horizontal line from this point to ogive, then a perpendicular line to cross x-axis, where it crosses the x-axis is the median.

iii) The mode represents the most typical value of a distribution and it should coincide with an existing item. The mode is not affected by the presence of extremely large or small items. It is not used extensively because of the following limitations:
- It is often not clearly defined
- Exact location is often uncertain
- It is unsuitable for further algebraic treatment
- It does not take into account extreme values

iv) The mean is the average of values being looked at.
The standard deviation is the square root of the average of square of deviations from the mean. In that case standard deviation cannot be obtained if the mean is not obtained first.

$$\sigma = \sqrt{\frac{(x - \bar{x})^2}{n}} \quad \text{Standard deviation}$$

$$\bar{x} = \frac{\sum x}{n} \quad \text{Mean}$$

Note:
- Notice the way to create continuity 0.5 is reduced from the lower class limit and added to the upper limit.
- Furthermore, notice the way the factor (i) 10^{-4} is removed and the assumed mean amount 0.97 is also removed. These are put back at the end in calculation of the mean.

<u>QUESTION 4</u>

a)

i) Quantity indices relate quantities in a given period to the quantity in another period (base period).
ii) Base year is the starting of a period to measure a given variable that changes with time.
iii) Chain index is where a given variable in one period is related to the same variable in the previous period.
iv) Retail price index is the index that relates prices of commodities from one period to another.

b)

Difference of 1999 salary from 1995 as:

$$\text{Stated terms} = \frac{\text{Salary of year of concern}}{\text{Salary of base year}} = \frac{\text{Salary of 1999}}{\text{Salary of 1995}} = \frac{400{,}000}{360{,}000} \times 100 = 111.1\%$$

$$\text{Constant} = \frac{\text{Deflated salary of year of concern}}{\text{Deflated salary of base year}} = \frac{\text{Deflated salary of 1999}}{\text{Deflated salary of 1995}} = \frac{269{,}905}{275{,}440} \times 100 = 98\%$$

There is a difference as shown hereby. There is a difference in percentage increase. In terms of stated shillings there is an increase of 11.1% while in terms of constant shillings, there is a decrease of 2 %
Although the rest of the years' changes were not required, it has been shown here for clear understanding

Year	Salary	CPI	%age change in terms of current shillings	Constant salary	%age change in terms of current shillings
1995	360,000	130.7	-	275,440	-
1996	370,000	136.2	102.8	271,659	98.6
1997	390,000	140.3	108.3	277,976	100.9
1998	395,000	144.5	101.3	273,356	99.2
1999	400,000	148.2	101.3	269,905	98

Constant salary (or deflated salary) = $\dfrac{\text{Salary of year of concern}}{\text{CPI}}$

For example for 1998 deflated salary = $\dfrac{\text{Salary of 1998}}{\text{CPI of 1998}} = \dfrac{395{,}000}{144.5}$ = Shs. 273,356

QUESTION 5

Month	Price	3 month	Weighted 3 month	Exponential α=0.35
Dec-00	40	-	-	-
Jan-01	38	-	-	-
Feb-01	39	-	-	39.300
Mar-01	41	39.000	38.800	39.195
Apr-01	36	39.333	39.600	39.827
May-01	41	38.667	38.600	38.487
Jun-01	34	39.333	39.000	39.367
Jul-01	37	37.000	37.200	37.488
Aug-01	35	37.333	36.600	37.317
Sep-01	37	35.333	35.600	36.506
Oct-01	40	36.333	36.200	36.679
Nov-01	41	37.333	37.800	37.841
Dec-01		39.333	39.800	38.947

a) March 2001 moving average

$$\frac{40+38+39}{3} = 39.0 = \frac{\text{Dec Price + Jan Price + Feb Price}}{3}$$

April 2001 moving average

$$\frac{38+39+41}{3} = 39.3 = \frac{\text{Jan Price + Feb Price + March Price}}{3}$$

So December 2001 will be $\dfrac{41+40+37}{3} = 39.33 = \dfrac{\text{Sep Price + Oct Price + Nov Price}}{3}$

b) March 2001

$0.4 \times 39 + 0.4 \times 38 + 0.2 \times 40 = 38.8 = 0.4 \times \text{Feb Price} + 0.4 \times \text{Jan Price} + 0.2 \times \text{Dec Price}$

April 2001

$0.4\times 41+ 0.4\times 39 + 0.2\times 38 = 39.6 = 0.4 \times \text{March Price} + 0.4\ \text{Feb Price} + 0.2 \times \text{Jan Price}$

So December 2001
$= 0.4 \times 41 + 0.4 \times 40 + 0.2 \times 37 = 39.8 = 0.4 \times$ Nov Price $+ 0.4 \times$ Oct Price $+ 0.2 \times$ Sept Price

Note:
This table is not necessary other than for the exponential smoothing method alone. The moving average and weighted moving average methods just need the last three months to forecast for December 2001.

c) Latest forecast = Previous forecast + α (latest observation – Previous forecast). e.g Feb = 40 + 0.35(39 – 40)
= 39.3

March = 39.3 + 0.35(39 – 39.3)
= 39.195

Note: That the January forecast was taken to be December 2000 sales since there was no previous month.

The forecast for December is obtained from the table as 38.947

d) The method I prefer is the exponential smoothing. This is because it incorporates all the data without cut off (like moving average), Greater weight is given to more recent data.

TOPIC 3

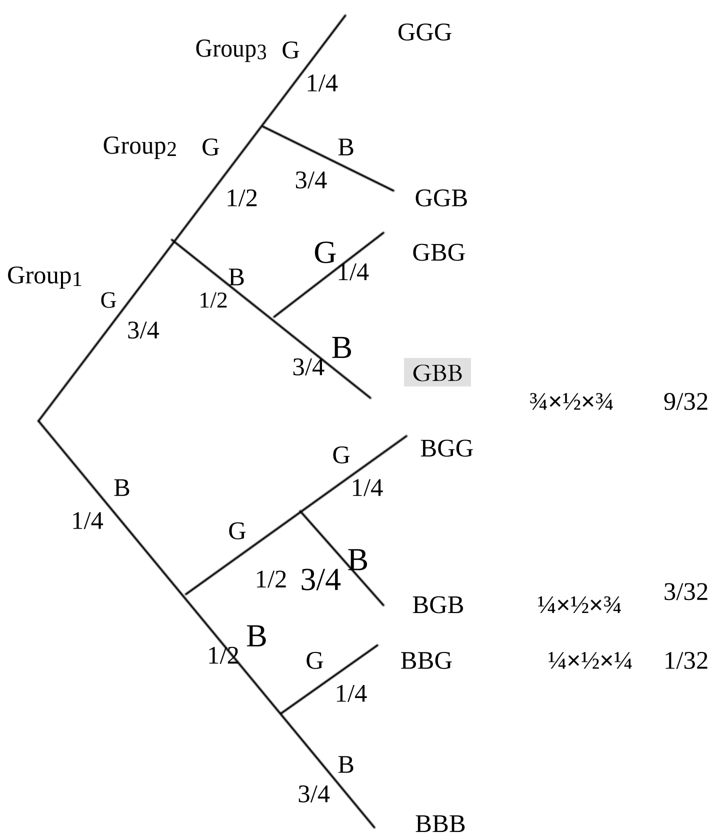
QUESTION 1

The product of the probabilities of each manager solving a problem gives probability of solving a problem. (since one manager solving a problem is independent of the others)

P (solving)= $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$

QUESTION 2

The best way to solve this is by use of a probability tree as follows:
Let G be the event of a girl being chosen
And B be the event of a boy being chosen



Sum of the required probabilities gives the following.

$P = \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$

QUESTION 3

i) Let A be the age group 30-40
B be the earnings more than 1500
Then P (B/A)= $\frac{P(AB)}{P(A)} = \frac{7/50}{12/50} = \frac{7}{12}$ Then the probability of B given A
Where: P(AB) - Probability of A and B occurring.
P(A) - Probability of A occurring.

ii) Let A be the age group below 50 years
B be the earnings varying between 2000-2500
Then P(B/A)= $\frac{P(AB)}{P(A)} = \frac{12/50}{20/50} = \frac{12}{20}$

QUESTION 4

Let A be the event drilling a well and B be the computer analysis showing an economic well. Then $P(AB) = P(A)P(B/A)$. But then since computer analysis and drilling of economic well are independent then $P(B/A) = P(B)$. So that $P(AB) = P(A)P(B) = 8\ 24 \times 0.8 = 0.267$

QUESTION 5

Let G-firms bid awarded
H-competitor submitting a bid

i) $P(G) = P(GH) + P(G\overline{H})$

$= P(H) \times P(G/H) + P(\overline{H}) \times P(G/\overline{H})$

$= 0.5 \times 0.3 + 0.5 \times \frac{3}{4} = 0.525$

Where $\overline{H}$ - event that competitor does not submit a bid

ii) $P(H/G) = \dfrac{P(GH)}{P(G)} = \dfrac{0.3}{0.525} = 0.571$

QUESTION 6

Let E, F, G, H- be items from plant 1,2,3,4 respectively and
D- be defective item

$P(G/D) = \dfrac{P(GD)}{P(D)}$

$P(D) = P(E) \times P(D/E) + P(F) \times P(D/F) + P(G) \times P(D/G) + P(H) \times P(D/H)$

$= 0.3 \times 0.05 + 0.25 \times 0.1 + 0.35 \times 0.15 + 0.1 \times 0.2$

$= 0.1125$

$P(GD) = P(G) \times P(D/G)$

$= 0.35 \times 0.25 = 0.0525$ So

$P(G/D) = \dfrac{0.0525}{0.1125} = 0.467$

QUESTION 7

a) Statistical independence of two events means that the probability of first event occurring given the second event has occurred is equal to the probability of second event occurring given the first event has occurred. That is, occurrence of one event does not affect the occurrence of the other event. Given two events A and B, $P(A/B) = P(B/A)$ for independent events.

b) Let A be -the event of the employee having accident in a particular year I be - the event of the employee being instructed in a particular year

i. The probability of employee being accident free given that he had no safety instructions is given by the following expression

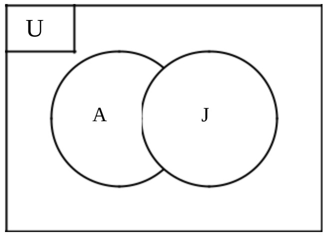
$P(\overline{A}/\overline{I}) = \dfrac{P(\overline{A}\overline{I})}{P(\overline{I})}$

Given; P(A)= 0.02 - probability of employees having a minor accident in a given year

P(I/A)= 0.3 - probability of employees having had safety instructions given they had accidents in a given year.

$P(\overline{I}) = 0.8$ - probability of employees not having safety instructions in a given year

Then $P(\bar{A}\bar{I})$ =1-(P(A) +P(I)- P(AI)) The shaded area shown here.



And P(AI)= P(I/A) P(A)=0.3×0.02=0.006

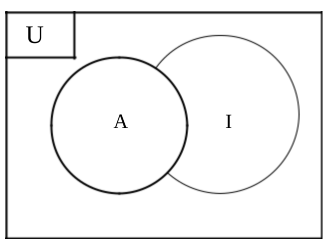
P(I) $= 1 - P(\bar{I}) = 1 - 0.8 = 0.2$

So $P(\bar{A}\bar{I})$ =1-(0.02+0.2-0.006)=0.786 and $P(\bar{A}/\bar{I}) = \frac{P(\bar{A}\bar{I})}{P(\bar{I})} = \frac{0.786}{0.8} = 0.9825$

ii. Probability of employee being accident free given that he had safety instructions is given by the following expression

$$P(\bar{A}/I) = \frac{P(\bar{A}I)}{P(I)}$$

$P(\bar{A}I)$ =(P(I) - P(AI)) The shaded area shown



And since P(AI) = 0.006 then

$P(\bar{A}I)$ = 0.2-0.006=0.194

And so $P(\bar{A}/I) = \frac{P(\bar{A}I)}{P(I)} = \frac{0.194}{0.2} = 0.97$

c) Poisson distribution is expressed by the following:

$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ Where x - event transformer being struck.

e-natural logarithm ≈ 2.718

λ -mean=0.4

i) The probability x = 0, $P(0) = \frac{4^0 e^{-0.4}}{0!} = e^{-0.4} = 0.6703$

ii) $P(x \le 2) = P(0) + P(1) + P(2)$

$$= 0.6703 + \frac{4e^{-0.4}}{1!} + \frac{4^2 e^{-0.4}}{2!} = 0.6703 + 0.2681 + 0.0536 = 0.9921$$

QUESTION 8

a) Given the proportion p is expected to be 0.48 and from past records $\hat{p}$ was found to be 0.4 and n = 10, then np = 4.8 and n(1-p) = 10(1-0.48) = 5.2.

Binomial distribution is appropriate for this event. $q = 1 - p = 1 - 0.48 = 0.52$

i) Probability of breaking the record is expressed as follows:

$$P(x > 4) = 1 - (P(0) + P(1) + P(2) + P(3) + P(4))$$

$$= 1 - \left({}^{10}C_0 p^0 q^{10} + {}^{10}C_1 p^1 q^9 + {}^{10}C_2 p^2 q^8 + {}^{10}C_3 p^3 q^7 + {}^{10}C_4 p^4 q^6 \right)$$

$$= 1 - q^{10} + \frac{10!}{1!9!} p^0 q^9 + \frac{10!}{2!8!} p^2 q^8 + \frac{10!}{3!7!} p^3 q^7 + \frac{10!}{4!6!} p^4 q^6$$

$$= 1 - \begin{array}{l} 0.52^{10} + 10 \times 0.48 \times 0.52^9 + 45 \times 0.48^2 \times 0.52^8 + 120 \times 0.48^3 \\ \times 0.52^7 + 210 \times 0.48^4 \times 0.52^6 \end{array}$$

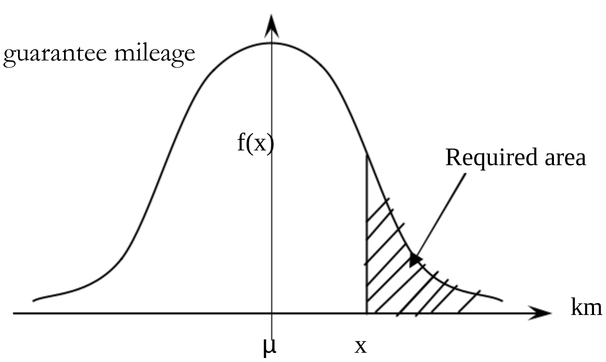
$$= 1 - 0.427 = 0.57$$

ii) The binomial distribution, because of the situation of success or failure with known probability. Sample size is small so that np<5 even though n(1-p)>5 slightly. Normal distribution could not be used.

b) Given $\mu = 36500$ km mean

$\sigma = 5000$km Standard deviation

i) x = 40000km guarantee mileage



$$Z = \frac{x - \mu}{\sigma} = \frac{40000 - 36500}{5000} = 0.7$$

From normal distribution tables:

$$P(x > 40000) = 0.5 - 0.258 = 0.242\ .$$

The company can distribute the tyres since P(x>40000 km) = 24.2% which is more than the required 20%.

ii) Given that $P(x < x_g) = 0.1$, from normal distribution table the Z value is 1.2817. So $Z = \frac{x_g - \mu}{\sigma} = \frac{x_g - 36500}{5000} = 1.2817$

$$\Rightarrow x_g = 5000 \times 1.21817 + 36500 = 42908 \text{ km}$$

c) The advantages of normal distribution (shows importance and uses too) include the following:

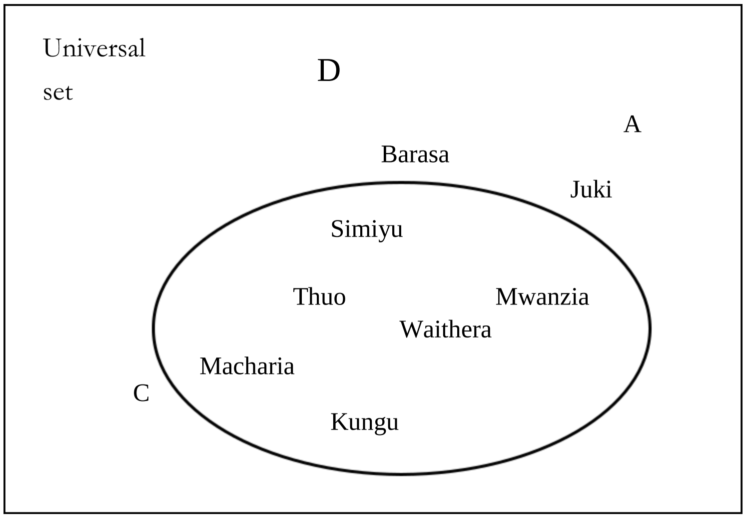
- It explains many physical characteristics like heights/weights of people and dimensions of items produced in a production process
- Normal distribution assists in sampling from determining the sample size to estimating parameters from statistics
- Is good for testing hypothesis and setting confidence limits
- It can be used to approximate other distributions which do not depart so much like binomial and poisson
- It is used in statistical quality control to set control limits

QUESTION 9

a) The sets can be presented as below:

i) A = {Simiyu, Juki, Waithera, Baraza, Thuo, Mwanzia}
C = {Simiyu, Macharia, Waithera,Kungu,Thuo, Mwanzia}
D = {Simiyu, Baraza,}

ii) The Venn diagram showing these set is as follows:



iii) The special relationship between sets A and D is that Baraza has both degree and diploma though not being a member of ACII.

iv) $A \cap C$ = {Simiyu, Waithera, Thuo, Mwanzia} These are the ones who have both a diploma and membership to ACII

$D \cup C$ = {Simiyu, Waithera, Thuo, Mwanzia, Baraza, Macharia, Kungu} This is a combination of those with degrees or membership to ACII or both.

$D \cap C$ = {Simiyu} The one with a degree and membership to ACII

v) A suitable universal set will be Insurance staff.

b) i) Error being not for consumable means it comes from either equipment or special order types.
So the probability of error not for being consumable is:

$$P = \frac{15}{60} + \frac{7}{60} = \frac{22}{60} = \frac{11}{30}$$

ii) Probability of the order coming from maintenance or production is:

$$P = \frac{10}{60} + \frac{7}{60} = \frac{27}{60}$$

iii) The probability of incorrect order of equipment coming from purchase is:

$$P(\text{Purchase}) \times P(\text{Equipment/Purchase}) = \frac{15}{60} \times \frac{3}{15} = \frac{3}{60} = \frac{1}{20}$$

c)

i) The condition for $P(A / B) = P(A)$ is that it is only possible only if the events A and B are independent. That means the occurrence of A does not affect the probability of occurrence of B.

ii) Addition rule states that, if an event can occur in more than one ways (all mutually exclusive), the probability of it happening at all is the sum of the probability of it happening in the several ways.

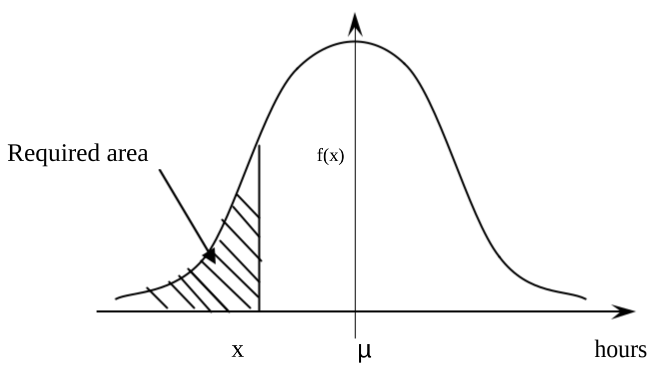
iii) Bayes theorem given two events A and B is as follows:

$P(A / B) = \frac{P(AB)}{P(B)}$ ⇒ Probability of occurrence of event A given that B has occurred is given by the probability of both events occurring divided by the probability of event B occurring.

QUESTION 10

a) i) Situation 1 is normal distribution. The lifetime is a continuous function, with the mean known and a standard deviation known. We can only know a given range and not a particular value.

ii)



Diagram

$Z = \frac{x-\mu}{\sigma} = \frac{600-800}{160} = -1.25$ from normal distribution table:

$P(x < 600) = 0.5 - 0.3944 = 1056\%$

iv) Given that $P(x < x_g) = 0.01$, from normal distribution table the Z value is 2.33.

So $Z = \frac{x_g - \mu}{\sigma} = \frac{x_g - 800}{160} = 2.33$

$\Rightarrow x_g = 160 \times 2.33 + 800 = 1172.8 \approx 1173$ hours

v) Given that the sample size n = 25, and mean of the sample $\bar{x} = 850$ hours then:

$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{850 - 800}{160/\sqrt{25}} = 1.5625$. From normal distribution table:

$P(\bar{x} > 850) = 0.5 - 0.4406 = 0.0594$

b)

i) This is a binomial distribution since it is accept/reject situation (have two events that are mutually exclusive) with respective value of mean that is known.

ii) Probability of purchasing a consignment is found as follows: Given that p = 0.15 then q = 1-0.15 = 0.85

$P(\text{unsatisfactory} \le 2) = P(0) + P(1) + P(2)$

$= \left({}^{10}C_0\, p^0 q^{10} + {}^{10}C_1 p^1 q^9 + {}^{10}C_2\, p^2 q^8\right)$

$= q^{10} + \frac{10!}{1!9!} p^1 q^9 + \frac{10!}{2!8!} p^2 q^8$

$= \left(0.85^{10} + 10 \times 0.15 \times 0.85^9 + 45 \times 0.15^2 \times 0.85^8\right) = 0.82$

c)

i) This is a Poisson distribution since there is a given rate, which is a mean for vehicles passing. In a large interval, the number of vehicles passing is scattered. The interval is small.

ii) The mean number of vehicles passing per 20 second interval is $2 \times \frac{20}{10} = 4$

The probability of more than 3 vehicles passing this point is expressed as follows;

$$P(x > 3) = 1 - \left(e^{-4} + \frac{e^{-4}4}{1} + \frac{e^{-4}4^2}{2} \right)$$

$$= 1 - 0.238 = 0.76$$

QUESTION 11

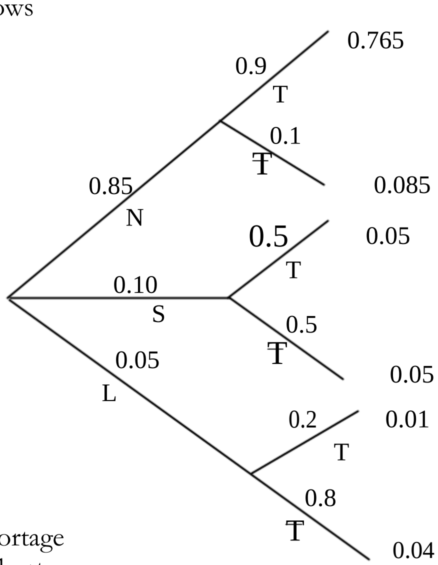
a) Probability is a measure of the likelihood of obtaining a particular outcome from an experiment. Given an experiment has n trials with no influence to each other, then having m outcomes of an event A, the probability of event A P(A)= m/n . It is between 0 and 1.

b) Bayes' theorem is as follows

$P(A / B) = \frac{P(AB)}{P(B)}$ Which is probability of occurrence of event A given that event B has occurred is given by the probability of occurrence of both events divided by the probability of occurrence of event B.

Bayes' theorem can be used to revise subjective probabilities made from beliefs. This is so when more information is added to what already exists.

c) The probability tree is as follows



Let event T- Passing of test
N-No shortage
S-Small shortage
L-Large shortage

i) Probability of failing test=0.085+0.05+0.04=0.175

Probability of having large or small inventory shortage given the failed test $= \frac{0.05 + 0.04}{0.175} = \frac{0.09}{0.175} = 0.514$

ii) The probability of passing test=0.765+0.05+0.01=0.825

Probability of no inventory shortage given the failed test $= \frac{0.765}{0.825} = 0.927$

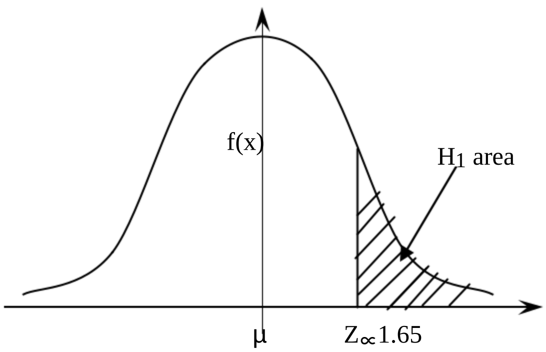
TOPIC 4

QUESTION 1

The null and alternative hypotheses are as follows:

H_0: $\mu = 5000$ Have same income

H_1: $\mu > 5000$ Have higher income



This is a one tailed test. Diagram

Given $\bar{x} = 5500$

$n = 144$

$\sigma = 1200$

$\alpha = 0.05$ then the test statistic is $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{5500 - 5000}{1200/\sqrt{144}} = 5$

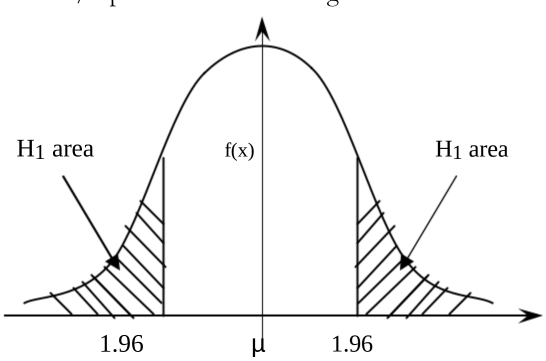
From normal distribution tables Z at α=0.05 is 1.65. Then, since calculated Z>1.65 then we reject H_0 accept H_1. Meaning the local workers have significantly higher income than the total population.

QUESTION 2

The null and alternative hypotheses are as follows:

H_0: $\mu = 15$ contact/representative has not changed

H_1: $\mu \neq 15$ contact/representative has changed



This is a two tailed test.

Given $\bar{x} = 16.5$

$n = 200$

$s = 3.5$

$\mu = 15$

$\propto = 0.05$ then the test statistic is $Z = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}} = \dfrac{16.5 - 15}{3.5/\sqrt{200-1}} = 6.046 > 1.96 \Rightarrow$ H_0 is rejected and H_1 is accepted. The average number of weekly sales contact per sales representative has changed significantly.

From normal distribution tables for two tailed test, Z at $\propto = 0.05$ is 1.96.

QUESTION 3

Calculation of $\bar{x}_1, s_1, \bar{x}_2, s_2$ is done first

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{1{,}048}{6} = 174.67$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{1{,}330}{7} = 190$$

Model A			Model B		
	x_1	$(x_1 - \bar{x}_1)^2$		x_2	$(x_2 - \bar{x}_2)^2$
1	180	28.44	1	195	25
2	176	1.78	2	194	16
3	184	87.11	3	190	0
4	181	40.11	4	192	4
5	190	235.11	5	187	9
6	137	1418.78	6	185	25
Total	1,048	1,811.33	7	187	9
				1,330	88

$$s_1 = \sqrt{\frac{\sum (x_1 - \bar{x}_1)^2}{n_1}} = \sqrt{\frac{1{,}811.33}{6}}$$

$= \sqrt{301.89}$

$= 17.37$

$$s_2 = \sqrt{\frac{\sum (x_2 - \bar{x}_2)^2}{n_2}} = \sqrt{\frac{88}{7}}$$

$= \sqrt{12.57}$

$= 3.55$

H_o $\quad \mu_1 = \mu_2$ means are equal

H_1 $\quad \mu_1 \neq \mu_2$ means are not equal

F-test

$$F = \frac{n_1 s_1^2}{n_1 - 1} \Big/ \frac{n_2 s_2^2}{n_2 - 1} = \frac{6 \times 301.89}{6 - 1} \Big/ \frac{7 \times 12.57}{7 - 1}$$

$= 24.7$

$F_{5\%} = 4.95$

$\Rightarrow$Variances are significantly different.

So:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}} = \frac{(174.67 - 190) - (0)}{\sqrt{\frac{301.89}{5} + \frac{12.57}{6}}}$$

So $t = \frac{-15.33}{7.90} = -1.94$

$t_{5\%} = -2.2$

Since calculated $t > t_{5\%} = -2.2$ then the H_o is accepted. That is, the two models have same productivity.

QUESTION 4

Observed frequencies

Room Charge.

Rating	**50**	**100**	**150**	**200**	**Total**
Excellent	10	25	28	28	91
Good	24	30	16	16	86
Average	80	82	21	13	196
Poor	40	41	18	25	124
Total	154	178	83	82	497

The expected frequencies= $\frac{\text{Total rating of a given charge}}{\text{Total rating}} \times \text{Total for a given rating}$

For example, expected frequency for excellent rating when the charge is 50 Rupees is

$\frac{154}{497} \times 91 = 28.197 \approx 28.2$

Expected frequencies

Room Charge

Rating	**50**	**100**	**150**	**200**	**Total**
Excellent	28.2	32.6	15.2	15.0	91
Good	26.6	30.8	14.4	14.2	86
Average	60.7	70.2	32.7	32.3	196
Poor	38.4	44.4	20.7	20.5	124
Total	154	178	83	82	497

The null and alternative hypotheses are as follows

H_0 : There is no difference in treatment according to room charges.
H_1 : There is a difference in treatment according to room charges.

A table to assist in calculation of $\chi^2 = \frac{\sum (f_o - f_e)^2}{f_e}$ is as follows.

Where f_e-expected frequency

f_o-observed frequency

fo	fe	$(fo - fe)^2/fe$
10	28.2	11.75
25	32.6	1.77
28	15.2	10.77
28	15.0	11.77
24	26.6	0.25
30	30.8	0.02
16	14.4	0.18
16	14.2	0.23
80	60.7	6.14
82	70.2	1.98
21	32.7	4.19
13	32.3	11.53
40	38.4	0.07
41	44.4	0.26
18	20.7	0.35
25	20.5	0.98
		61.74

For $(4-1)\times(4-1)$ degrees of freedom and 0.05 level of significance

$$\chi = 16.9$$

Since calculated χ is greater than $\chi_{0.059} = 16.9$ then H_o is rejected. So there is a difference in treatment to customers according to rate charged.

QUESTION 5

a) One-sided confidence interval is the limit of one side of a distribution. It is usually necessary when testing something above or lower side of the parameter of population given. The parameter can be mean or variance.

b) The decision rule is not clear-cut. The null hypothesis should indicate the decision point, and not a region as is indicated by $\bar{x} \leq 4.7$.

c) Since hypothesis is usually a belief about a certain aspect different levels of type 1-error can be set depending on ones outlook of error and cost. (Adverse, neutral or risk taker).

d)

i) $S_p^2 = \dfrac{9\times 2.32^2 + 9\times 2.18}{10+10-2} = 5.0674$ and

$$t = \frac{(16-4)-0}{\sqrt{5.0674\left(\frac{1}{10}+\frac{1}{10}\right)}} = \frac{12}{1.0067} = 11.91 > t_{0.005,\,18} = 2.88$$

So, reject H_o ⇒Yes. One can conclude that successful and unsuccessful records differ with respect to mean amount of airplay.

ii) The assumption made is that the two means of the population are equal.

QUESTION 6

a) Given proportions $\hat{P}_1 = 0.35$, $\hat{P}_2 = 0.4$, and $n_1 = n_2 = 400$, the hypothesis is as follows.

H_o : $P_1 = P_2$ Campaign did not increase sales.

H_1 : $P_1 < P_2$ Campaign increased the sales.

At 5% level of significance ($Z_{0.05} = 1.65$)

Note: If one is not given the level of significance, then you can choose 5% which is not very strict or very tight.

The test statistic chosen is

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}}$$

$$\hat{q}_1 = 1 - \hat{P}_1 = 1 - 0.35 = 0.65$$

$$\hat{q}_2 = 1 - \hat{P}_2 = 1 - 0.4 = 0.6$$

So, $Z = \dfrac{(0.35 - 0.4) - 0}{\sqrt{\frac{0.35\times 0.65}{400} + \frac{0.4\times 0.6}{400}}} = \dfrac{-0.05}{0.034} = -1.46 > Z_{0.05} = -1.65$

So, the null hypothesis is accepted. This means that the campaign did not increase sales

b) H_o : $\mu = 1196$ New machine does not affect production

H_1: $\mu \neq 1196$ New machine affects production

Given $n = 12$; $S = 71$; $\bar{x} = 1158$. Then

The test statistic $Z = \dfrac{\bar{x} - \mu}{S/\sqrt{n}} = \dfrac{1158 - 1196}{71/\sqrt{12}} = -0.155$

At 5% level of significance $Z_{0.025} = -1.96$

Since calculated $Z = -0.155 > Z_{0.025} = -1.96$, then the null hypothesis is accepted. That is, the new machine does not affect production.

c)

i) - Age distribution of students in a school.
- Grades of a given examination.
- Lifetime of electrical goods.
- Production from a given machine.

ii) Normal distribution can be used as an approximation to the binomial distribution when the number considered is large, n>30, np (product of number and probability of success) and nq (product of number and probability of failure) are greater or equal to 5.

iii) Confidence limits measures the extent that we are sure a particular value of parameter lies.

QUESTION 7

a)

i) Random variable is family income in Shs. Y for a given family x. This is a discrete variable.
ii) Family size in number z for a given family x. This is a discrete variable.
iii) Distance from home to store site w in kilometers for a given family x. This is a continuous variable.
iv) Own dog/cat or not for a given family x. This is discrete.

b)

Day	N	σ	Lower z_1	p	Upper z_2	p	Required probability
Monday	120,000	20,000	-0.5	0.1915	1.5	0.4332	0.6247
Tuesday	100,000	25,000	0.4	0.1554	2	0.4772	0.3218
Wednesday	100,00	10,000	1	0.3413	5	0.5	0.1587
Thursday	120,000	40,000	-0.25	0.0987	0.75	0.2734	0.3721
Friday	140,000	20,000	-1.5	0.4332	0.5	0.1915	0.6247
Saturday	160,000	50,000	-1	0.3413	-0.2	0.0793	0.262

The lowest probability that the store will sell between Sh.110,000 and Sh.150,000 is 0.1587 occurring on Wednesday.

Lower limit

$$z_1 = \frac{x_1 - \mu}{\sigma} \qquad z_2 = \frac{x_2 - \mu}{\sigma}$$

x_1, x_2 – lower and upper limit respectively

$x_1 = 110$ $\qquad$ $x_2 = 150$

μ_d, σ_d - mean and standard deviation of the days sales.

z_1, z_2 – the standard normal variable as lower and upper limit.

Required probability is

$P(z_2) + P(z_1)$ depending on the side that x_1, x_2 lie on against μ.

Reading from normal distribution table, (usually given in exams), then –ve sign is used if x_1, and x_2 lie on the same side and the +ve sign is used if the x_1 and x_2 lie on different sides.

c) This is a case of hypothesis testing

H_0: $p = 30\%$ So no need to issue a warning letter.

H_1: $p > 30\%$ Issue a warning letter.

$\bar{p} = 0.2$, $p = 0.3$, $n = 5$

then $np = 0.3 \times 5 = 1.5 < 5$

$n(1-p) = 0.7 \times 5 = 3.5 < 5$ so t test is appropriate

t test:

$$t = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.2 - 0.3}{\sqrt{\frac{0.3(1-0.3)}{5}}} = \frac{-0.1}{\sqrt{\frac{0.3 \times 0.7}{5}}} = -0.488$$

Given 10% level of significance

$t_{10\%, 5} = 1.48 >$ calculated $t = -0.488$ so we accept the null hypothesis and the warning letter should not be issued.

TOPIC 5

QUESTION 1

a) $y = a + bx$

Where a and b are determined as follows

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

So given that $\sum x = 66$, $\sum y = 212.1$, $\sum x^2 = 506$, $\sum xy = 1{,}406.7$, $\sum y^2 = 4{,}254.08$ x = number of years, y = annual profit

Then $b = \frac{11 \times 1406.7 - 66 \times 212.1}{11 \times 506 - (66)^2} = 1.219$

And $a = \frac{212.1}{11} - 1.219 \times \frac{66}{11} = 11.967$

So $y = 11.967 + 1.219 \times x$

b) 12th year profit $y_{12} = 11.967 + 1.219 \times 12 = 26.595$

13th year profit $y_{13} = 11.967 + 1.219 \times 13 = 27.814$

c) $$r^2 = \frac{\left(n\sum xy - \sum x \sum y\right)^2}{\left(n\sum x^2 - (\sum x)^2\right) - \left(n\sum y^2 - (\sum y)^2\right)}$$

$$r^2 = \frac{\left(11 \times 1406.7 - 66 \times 212.1\right)^2}{\left(11 \times 506 - 66^2\right) - \left(11 \times 4251.08 - 212.1^2\right)}$$

$r^2 = 0.9944$

99.44% of the variation in annual profit can be predicted by change in actual values of numbers of years.

d) $S_e = \sqrt{\frac{\sum y - a\sum y - b\sum xy}{n-2}} = \sqrt{\frac{(y-\hat{y})^2}{n-1}} = \sqrt{\frac{0.916}{9}} = 0.319$

Given 95% confidence interval for the line, at 9 degrees of freedom the t value is $t_{95\%,9} = 2.2622$ The confidence interval for the regression line is:

$$y \pm t_{95\%} \times S_e\sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum x^2 - \frac{(\sum x)^2}{n}}} \text{ and given } \bar{x} = \frac{\sum x}{n} = \frac{66}{11} = 6$$

$$y \pm 2.2622 \times 0.319\sqrt{\frac{1}{11} + \frac{(x-6)^2}{506 - \frac{(66)^2}{11}}}$$

$$y \pm 0.722\sqrt{\frac{1}{11} + \frac{(x-6)^2}{110}}$$

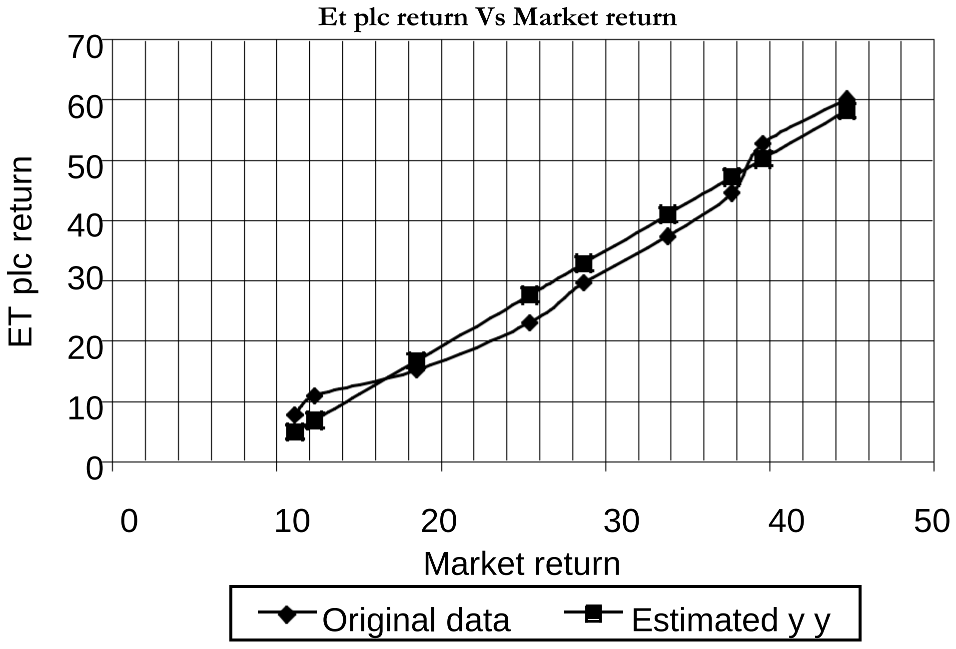
e) The regression line will be a good estimator of profit because r^2 was high (meaning that variation in profit can be highly explained by actual number of years). The standard error of regression line was also very small.

QUESTION 2

a.

Year	ET plc return y_i	Group return x	Estimated y y	$(y_i-y)^2$
1	7.8	11.1	4.97	8.01
2	11	12.3	6.88	16.89
3	15.2	18.5	16.73	2.3
4	23.1	25.4	27.69	21.07
5	29.7	28.7	32.94	10.5
6	37.4	33.8	41.04	13.25
7	44.6	37.7	47.24	6.97
8	52.8	39.6	50.26	6.45
9	60.2	44.7	58.36	3.39

The graph will look as follows'



Given $\sum y = 345.7$

$\sum x = 297.3$

$\sum y^2 = 15711.59$

$\sum x^2 = 10285.83$

$\sum xy = 12577.02$

Then $b = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$

And $a = \dfrac{\sum y}{n} - b\dfrac{\sum x}{n}$

so $b = \dfrac{10 \times 12577.02 - 297.3 \times 345.7}{10 \times 10285.83 - (297.3)^2}$

$b = 1.589$

$a = \dfrac{345.7}{10} - 1.589 \times \dfrac{297.3}{10}$

$= -12.669$ b. So $y = -12.669 + 1.589x$

$$S_e = \sqrt{\sum \frac{(y_i - y)^2}{n-2}} = \sqrt{\frac{107.18}{8}} = 3.66$$

$$S_b = \frac{S_e}{\sqrt{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}} = \frac{3.66}{\sqrt{10285.83 - 10 \times \left(\frac{297.3}{10}\right)^2}} = 0.1$$

So the confidence limit of b given $t_{95\%,8} = 2.31$

$b = 1.589 \pm 2.31 \times 0.1 = 1.589 \pm 0.22$

c. The line made from the least square method, best fits the data even the variation of slope is very small.

QUESTION 3

a) $t = \dfrac{b_i - 0}{S_{b_i}} = \dfrac{\text{slope}_i}{\text{standard error of slope}}$

Confidence interval $= b_i \pm t_{0.975\%,n-1-3} S_{b_i}$

$t_{0.975,24-1-3} = 2.09$

	Calculated t	Confidence interval
For X_1:	$t = \dfrac{0.021}{0.019} = 1.11$	$0.021 \pm 2.09 \times 0.019 = 0.021 \pm 0.04$
X_2:	$t = \dfrac{0.075}{0.034} = 2.206$	0.075 ± 0.071
X_3:	$t = \dfrac{0.043}{0.018} = 2.389$	0.043 ± 0.038

b) The assumptions include:
- Error or residuals are independent and normally distributed for a given value of x.
- Expected value of error is equal to zero
- Variance of errors is the same for all x's.

These assumptions are set up to enable one to come up with a projection of the population from the sample. So they are reasonable.

c) X_1 gives the strongest evidence of being statistically discernible because the t statistic calculated is within the required range.

d) The decision to keep or drop the first regressor will be based not only on t-test, but also looking at the r^2 and standard error of the regression in general. The main objective is to include the regressor that reduces standard error of regression and r^2 value is large. Other than just having the t test alone. In this case since t calculated is within the required range and standard error of regression is low, then it will be appropriate to include the first regressor x_1 in the final regression

QUESTION 4

a) Not finding a linear relationship does not necessarily mean that a relationship does not exist. Other relationships may exist that are non-linear. May be logarithmic, exponential or quadratic.
Linear relationship is of the form $y = a + bx_1 + cx_2$ for a 2 variable for example.

b) Correlation measures the direction and extent one variable (dependent) is affected by another variable (independent). So high correlation means the independent variable causes the dependent variable to vary.

c) Given that x=30 then:

i) $\hat{y} = 27.32 + 1.3 \times 30 = 66.32$

ii) The relationship is linear with a given value of 27.32 even without exposure to insecticides. This value of y increases for any hour of exposure to insecticide by a factor of 1.3.

iii) Coefficient of determination r^2=0.86

H_o: r = 0 A relationship exists

H_1: $r \neq 0$ A relationship does not exist.

$$t = \sqrt{\frac{r^2(n-2)}{(1-r^2)}} = \sqrt{\frac{0.86(16-2)}{1-0.86}} = 9.27 > t_{0.975\%,14} = 2.14$$

So we reject H_0 and accept H_1 that a relationship actually exists

iv) Assumptions include:

- Relationship is linear
- Independent variable x is known, so used to predict y
- Errors are normally distributed with expected value of zero for any value of x
- Variance of errors is a constant
- Errors are independent

Note: Test statistic is distributed as student's t with n – 2 degrees of freedom and is given by:

$t = {}_{r;\, n-2;} = 2.14$ from t-tables

QUESTION 5

a) Let y be enrolment and x be quarter of a year. Then $y = a + bx$ where

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} \text{. So } b = \frac{20 \times 22253 - 210 \times 1784}{20 \times 2870 - (210)^2} = 5.29$$

And $a = \frac{1784}{20} - 5.29 \times \frac{210}{20} = 33.61$ giving the expression for y as follows: $y = 33.61 + 5.29 \times x$.

1996	Quarter x	$y = 33.61 + 5.29 \times x$
First	21	144.7
Second	22	150.0
Third	23	155.3
Fourth	24	160.6

There is an overall trend of increased enrolment with time. Other than the seasonal variation, the relationship can be seen to be linear. So the regression equation is appropriate.

b) The other factors to be included are income, level of education and population growth.

c) The expected enrolment will be followed as a general trend with seasonal variations. Justification of calculation.

$$r^2 = \frac{\left(n\sum xy - \sum x \sum y\right)^2}{\left(n\sum x^2 - \left(\sum x\right)^2\right)\left(n\sum y^2 - \left(\sum y\right)^2\right)}$$

$$r^2 = \frac{20 \times 22253 - 210 \times 1784}{20 \times 2870 - (210)^2 - \left(20 \times 211254 - 1784^2\right)}$$

$$r^2 = 0.77$$

And r=0.6 meaning there is a positive correlation and 77% of the variation is explained by the quarters.

Calculation of $\sum y^2$

Quarter	Enrolment y	y^2
1	30	900
2	42	1764
3	100	10000
4	66	4356
5	32	1024
6	107	11449
7	71	5041
8	47	2209
9	41	1681
10	93	8649
11	139	19321
12	62	3844
13	45	2025
14	101	10201
15	151	22801
16	67	4489
17	73	5329
18	181	32761
19	227	51529
20	109	11881
Sum		211254

QUESTION 6

a) Regression analysis checks the study of relationship between variables (independent and dependent). Correlation analysis on the other hand looks at the strength of relationship of variables. Both regression and correlation analysis are related since they both look at variables behaviour to each other.

b) The regression line will be:

i) ITE_t=-52.59-0.20PR_t-1.48$PROB_t$

ii) For any coefficient of the independent variables that is above the t value at o.o5 significance level is eliminated.

Note: This should not be followed as a rule generally. Here it has been used since there in no much information about the standard error of regression and the correlation coefficient. Otherwise the main aim is to include the variables that give the least standard error and highest coefficient of determination.

iii) The –52.59 constant term indicates that people usually pay up their taxes without evading. The 33.44 coefficient of true income indicated that people would tend to evade tax more when their income is increased. The –1.48 coefficient of probability of detection indicates that people will tend to reduce evasion when they realize there are higher probabilities of being detected.

iv) From the regression results, it is true, that it had a negative influence on income tax evasion.

QUESTION 7

		x	A=y	Quarterly moving average	Centred moving average T	x2	xy	A / T	Deseasonalised values A / S
1983	3	1	12			1	12		11.06
	4	2	16			4	32		8.122
				8.5					
1984	1	3	5		8.25	9	15	0.6	6.748
				8.0					
	2	4	1		8.125	16	4	0.123	4.902
				8.25					
	3	5	10		8.5	25	50	1.176	9.217
				8.75					
	4	6	17		8.75	36	102	1.943	8.629
				8.75					
1985	1	7	7		8.75	49	49	0.800	9.447
				8.75					
	2	8	1		8.625	64	8	0.116	4.902
				8.5					
	3	9	10		8.75	81	90	1.143	9.217
				9.0					
	4	10	16		9.25	100	160	1.730	8.122
				9.5					
1986	1	11	9		9.25	121	99	0.973	12.146
				9					
	2	12	3		9.25	144	36	0.324	14.706
				9.5			0		
	3	13	8		9	169	104	0.889	7.373
				8.5					
	4	14	18		8.375	196	252	2.149	9.137
				8.25					
1987	1	15	5		9.125	225	75	0.548	6.748
				10					
	2	16	2		8.375	256	32	0.239	9.804
				6.75					
	3	17	15			289	255		13.825
	4	18	5			324	90		2.538
Total		171	160			2109	1465		

Approximating the trend to be linear, then

Trend line - T = a + b ×Quarter number.

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$b = \frac{(n\sum xy - \sum x \times \sum y)}{n\sum x^2 - (\sum x)^2}$$

given that
$\sum x = 171$
$\sum x^2 = 2109$
$\sum y = 160$
$\sum xy = 1465$
$n = 18$

$$b = \frac{(18 \times 1465 \times 171 - 171 \times 160)}{18 \times 2109 - 171^2} = -0.1135$$

$$a = \frac{\sum y}{n} - b \times \frac{\sum x}{n} = \frac{160}{18} - (-0.1135) \times \frac{171}{18} = 9.9673$$

So, T = 9.9673-0.1135 × Quarter number
Notes:
Any number of years moving average can be used. Quarterly moving average has been chosen in this case. Since it is not centred, centering is done as shown.
The trend can also be obtained from the time series as required here.
The summation for quarter numbers and the actual production are obtained. The additional values of summation of x^2 (quarter number squared) and summation of xy (production × quarter number) are obtained from the additional columns indicated.
The values of a and b of the trend line equation can then be obtained as shown.
Though not required;
It was possible to obtain deseasonalised data before obtaining the tend line. This means a better forecasting equation is obtained (moving average and trend equation would have been used)
Seasonal factor S is obtained by averaging the error variation A/T for each quarter as per the second table. Since the summation of the average is not equal to 4 (seasonal aspect) it has to be corrected by the factor 4/3.941.
The deseasonalised data is then obtained. Notice the way here a multiplicative model was chosen because of the way the seasonal aspect keeps on changing.
Determination of S

	1	2	3	4	
1983	-	-	-	-	
1984	0.6	0.123	1.176	1.943	
1985	0.8	0.116	1.143	1.730	
1986	0.973	0.324	0.889	2.149	
1987	0.548	0.239	-	-	Total
Average	0.730	0.201	1.069	1.941	3.941
Corrected	0.741	0.204	1.085	1.970	

So the summations will change to be as follows
$\sum x = 171$
$\sum x^2 = 2109$
$\sum y = 156.643$
$\sum xy = 1507.171$
$n = 18$

$$b = \frac{(18 \times 1507 \times 171 - 171 \times 156.643)}{18 \times 2109 - 271^2} = 0.0393$$

$$a = \frac{\sum y}{n} - b \times \frac{\sum x}{n} = \frac{156.643}{18} - 0.0393 \times \frac{171}{18} = 5.1556$$

So, T = 5.1556 + 0.0393 × Quarter number

QUESTION 8

a)

i)

Month	No. (t)	Jobs Y	t^2	Yt
March	1	353	1	353
April	2	387	4	774
May	3	342	9	1026
June	4	374	16	1496
July	5	396	25	1980
August	6	409	36	2454
September	7	399	49	2793
October	8	412	64	3296
November	9	408	81	3672
Total	45	3480	285	17844

$$b - \text{slope} = \frac{n\sum tY_t - \sum t \sum Y_t}{n\sum t^2 - \left(\sum t\right)^2} = \frac{9 \times 17844 - 45 \times 3480}{9 \times 285 - 45^2} = 7.4$$

$$a - \text{intercept} = \frac{\sum y}{n} - b \times \frac{\sum x}{n} = \frac{3480}{9} - 7.4 \times \frac{45}{9} = 349.7$$

With least square method

T- trend = a + b ×No. of month.

Given a = 349.7 and b = 7.4, then T = 349.7 + 7.4 × No. of month.

December ⇒ No. of month is 10,

So the forecast is

T = 349.7 + 7.4 × 10

= 423.7 repair jobs

≈ 424 repair jobs.

ii)

Month	Jobs	Weighted 3 month moving average
March	353	-
April	387	-
May	342	-
June	374	356.6
July	396	365.7
August	409	384
September	399	401.6
October	412	401.7
November	408	407.8
December		408.3

December's forecast here is 408 jobs. This is lower than that approximated by least square method of 424 jobs.

Note: June $= 0.6 \times 342 + 0.3 \times 387 + 0.1 \times 353$
$= 0.6 \times$ May jobs $+ 0.3 \times$ April jobs $+ 0.1 \times$ March jobs

b)

i) Time series analysis is a method of studying the behaviour of variables changing with time. The main aim is to come up with a given pattern to enable one forecast the variable in a future period.

ii) Season is a pattern formed within a given period over the trend (which is the general change over time). For example the change from winter to summer in a given year will explain a season in temperature readings / rain in any given year.

Another example is the consumption of alcohol during the month, starting a lot then reducing during the mid-month and then increasing at end-month.

iii) The method of semi-averages uses two averages to obtain a trend from a given data. Firstly the variable values are divided into two halfway. The first half number of variables are averaged and the average put at the centre of them. Second half number variables are also averaged and the average put at the centre of them.

Secondly, the trend line is drawn by joining the two average points.

iv) Forecasts must be treated with caution because of the following:
- Since historical data is used; it may not necessarily mean that is expected in the future.
- Furthermore, the extension cannot be pushed far ahead from the last historical data -Accuracy reduces.

TOPIC 6

QUESTION 1

a) Formulation of problem in standard form. $z - 7x_1 - 5x_2 - 0s_1 - 0s_2 = 0$

$5x_1 + 3x_2 + 1s_1 + 0s_2 = 50$

$4x_1 - 2x_2 + 0s_1 + 1s_2 = 30$

I Basic solution

Basis	x_1	x_2	s_1	s_2	Solution	Ratio	
s_1	5	3	1	0	50	10	
s_2	4	-2	0	1	30	7.5	⇐
z	-7	-5	0	0	0		
	⇑						
II							
s_1	0	11/2	1	-5/4	12.5	2.27	⇐
x_1	1	-1/2	0	1/4	7.5	-15 Ignore	
z	0	-17/2	0	7/4	52.5		
		⇑					
III							
x_2	0	1	2/11	-5/22	2.27	-9.99 ignore	
x_1	1	0	1/11	3/22	8.63	63.29	⇐
z	0	0	17/11	-2/11	71.8		
				⇑			
IV							
x_2	5/3	1	1/3	0	16.65		
s_2	22/3	0	2/3	1	63.29		
z	2	0	19/11	0	89.06		

Stop the iteration here since the z function row has no more negative coefficients.

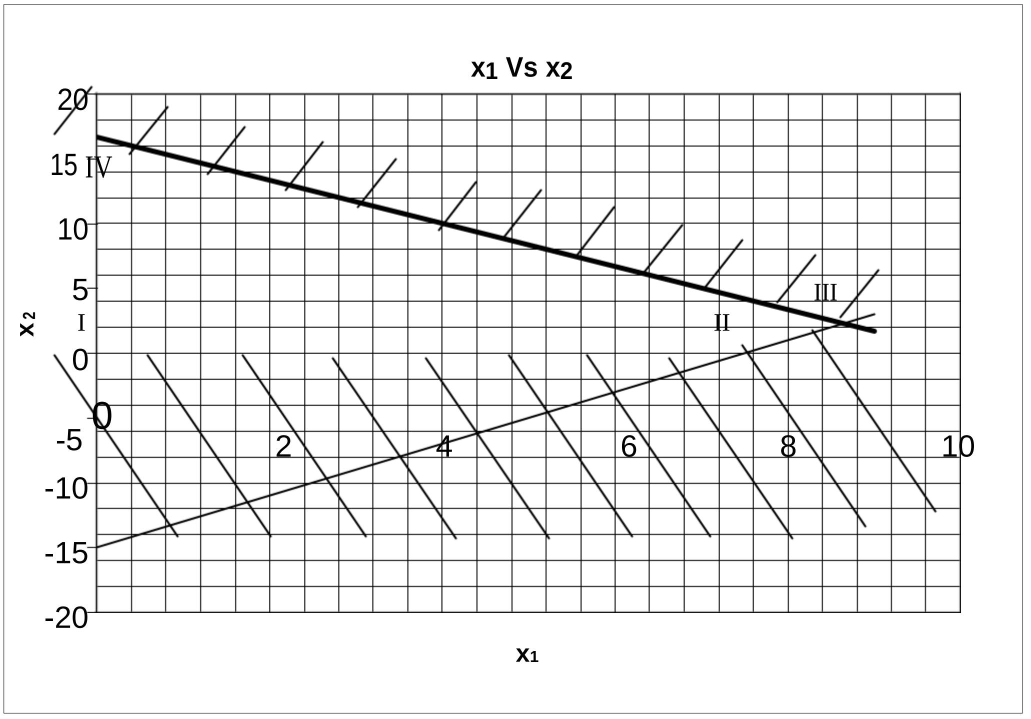
Solution. x_2 = 16.65 meaning produce 16.65 units of x_2.

s_2 = 63.29 meaning amount of slack for resources 2.

x_1 = 0 meaning do not produce x_1.

s_1 = 0 meaning resource 1 is used up.

b) Graph is as follows.



The various corner points in relation to the tableaus are indicated on the graph.
I (0,0), II(7.5,0), III(8.63,2.27) and IV(0,16.65)

c) Shadow prices for the problem are

$s_1 = 19/11$ If one unit of resource 1 is added then the z function will increase by 19/11.That means we can pay up to 19/11 to add resource 1.

If we produce any unit of x_1 then profit will reduce by 2 units.

QUESTION 2

a) Let a, b, and c, be number of machines A, B, and C. These are the decision variables. Formulation of LP model

Maximise

Output $U = 250a + 260b + 225c$

Subject to the constraints.

Capital budget	$25a + 30b + 22c \leq 350$	£ '000'
Floor space	$200a + 250b + 175c \leq 4000$	Square feet
	$a, b, c \geq 1$	

b) Linear / Proportion – the number of units with capital budget and floor space are linearly related.

Deterministic – the coefficients for the variables and constraints are known with certainty.

Additive – Buying one more of a given machine gives more production or additional production. Effect is additive.

Divisible – This requires that the machines and given constraints to be divisible. In this case the assumption does not hold. Here we have to take a machine as a whole and not ½ or ¼ or fraction of the machine.

c) Computer solution and analysis. Target Cell (Max)

Name	Original Value	Final Value
zfunc sol	0	3527.045455

Adjustable Cells

Name	Original Value	Final Value
sol a	0	1
sol b	0	1
sol c	0	13.40909091

Constraints

Name	Cell Value	Status	Slack
capbudget '000' sol	350	Binding	0
flospace (sq ft) sol	2796.590909	Not Binding	1203.409
sol a	1	Binding	0
sol b	1	Binding	0
sol c	13.40909091	Not Binding	12.40909

Adjustable Cells

Name	Final Value	Reduced Gradient
sol a	1	-5.681818182
sol b	1	-46.81818182
sol c	13.40909091	0

Constraints

Name	Final Value	Dual Price
capbudget '000' sol	350	10.22727273
flospace (sq ft) sol	2796.590909	0

Target Name	Value
zfunc sol	3527.045455

Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
sol a	1	1	3527.04	1	3527.04
sol b	1	1	3527.045	1	3527.04
sol c	13.40909091	1	735	13.40909091	3527.04

The solution of the problem is as follows
The number of machines to buy is:
A=1
B=1
C=13
The maximum output is 3527 pieces per day.
The capital budget will be used up completely and the floor space will be having a slack of 1203 square feet. So the dual price of the capital budget is $10.22.

Note: In exams, the solution from a computer package will be given and the student will be required to interpret the solution

QUESTION 3

Let x_1, x_2, x_3, x_4, x_5, and x_6 be the amount invested for the 3-year period for alternatives 1, 2, 3, 4, 5, and 6. Then the objective function will be:

z – Total dollar return.

Maximize $z = 0.3x_1 + 0.34x_2 + 0.35x_3 + 0.37x_4 + 0.38x_5 + 0.45x_6$

Subject to the constraints:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 300{,}000$ $ Capital budget.

$x_2 \geq 50{,}000$ $ min limit of alternative 2

$x_5 \leq 40{,}000$ $ max limit of alternative 5

$x_4 + x_6 \leq 75{,}000$ $ max limit of total of 4 and 6

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Effective Annual rate of return

Investment	Year 1	Year 2	Year 3	Total
1	0.12	0.10	0.08	0.3
2	0.14	0.10	0.10	0.34
3	0.15	0.12	0.08	0.35
4	0.10	0.12	0.15	0.37
5	0.08	0.12	0.18	0.38
6	0.25	0.15	0.05	0.45

Computer solution and analysis.

Target Cell (Max)

Name	Original Value	Final Value
return sol	0	113200

Adjustable Cells

Name	Original Value	Final Value
x1	0	0
x2	0	50000
x3	0	135000
x4	0	0
x5	0	40000
x6	0	75000

Constraints

Name	Cell Value	Status	Slack
capbudget	300000	Binding	0
limitalt2	50000	Binding	0
limitalt5	40000	Binding	0
limitalt4&6	75000	Binding	0
x1	0	Binding	0
x2	50000	Not Binding	50000
x3	135000	Not Binding	135000
x4	0	Binding	0
x5	40000	Not Binding	40000
x6	75000	Not Binding	75000

Name	Final Value	Reduced Gradient
x1	0	-0.0499
x2	50000	0
x3	135000	0
x4	0	-0.0801
x5	40000	0
x6	75000	0

Name	Final Value	Dual Price
capbudget sol	300000	0.35
limitalt2	50000	-0.01
limitalt5	40000	0.03
limitalt4&6	75000	0.1

Target Name	Value
Return sol	113200

Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
sol x1	0	0	113200	0	113200
sol x2	50000	50000	113200	50000	113200
sol x3	135000	135000	113200	135000	113200
sol x4	0	0	113200	0	113200
sol x5	40000	40000	113200	40000	113200
sol x6	75000	75000	113200	75000	113200

The present amount value to be invested in each of the alternatives is:
Alternative 1- $0
Alternative 2- $50,000
Alternative 3- $135,000
Alternative 4- $0
Alternative 5- $40,000
Alternative 6- $7,5000
There is no slack in any of the constraints. The dual prices are:

Capital budget	$0.35
Limitation on alternative 2	$0.01
Limitation on alternative 5	$0.03
Limitation on alternative4 and 6	$0.1

Note: Even though solution was not requested, it has been presented to add on intepretation of LP computer analysis.

b) **Assumptions:**

- Proportional – Increase in more investment ⇒ more return.
- Deterministic – Estimates of present value for each alternative (though estimated).
- Additive – To make up total dollar return, the investment return are added together.
- Divisible – Fractional amounts of the investment options are possible.

QUESTION 4

Let x_1, x_2, x_3, x_4, x_5, and x_6 be the number of shares invested in alternatives 1, 2, 3, 4, 5, and 6. And z be the total return (growth and dividend) for the investment. Then the objective function will be:

Portfolio data

	Alternative					
	1	2	3	4	5	6
Current price per share	\$80	\$100	\$160	\$120	\$150	\$200
Projected annual growth rate	0.08	0.07	0.10	0.12	0.09	0.15
Annual growth (rate × Current price)\$	6.40	7.00	16.00	14.40	13.50	30.00
Projected annual dividend per share\$	4.00	6.50	1.00	0.50	2.75	0.00
Total return per share in \$	10.10	13.50	17.50	14.90	16.25	30.00
Total risk (risk rate × Current price)\$	4	3	16	24	9	16

$$z = 10.4x_1 + 13.5x_2 + 17.5x_3 + 14.9x_4 + 16.25x_5 + 30x_6$$

Constraints are:

(1) $200x_6 \leq 250{,}000$ Maximum amount for alternative 6 in \$

(2) $80x_1 + 100x_2 \leq 500{,}000$ Maximum amount for alternatives 1 and 2 combined in \$

(3) $$\frac{4x_1 + 3x_2 + 16x_3 + 24x_4 + 9x_5 + 16x_6}{80x_1 + 100x_2 + 160x_3 + 120x_4 + 150x_5 + 200x_6} \leq 0.10$$

$\Rightarrow 4x_1 + 7x_2 + 0x_3 - 12x_4 + 6x_5 + 4x_6 \geq 0$ Maximum total risk in \$

(4) $x_1, x_2, x_3, x_4, x_5, x_6 \geq 100$ Non-negativity and Minimum number of shares of each alternative

(5) $80x_1 + 100x_2 \geq 250{,}000$ Minimum amount for alternatives 1 and 2 combined in \$

(6) $4x_1 + 6.5x_2 + x_3 + 0.5x_4 + 2.75x_5 \geq 10{,}000$ Minimum dividend in \$

(7) $80x_1 + 100x_2 + 160x_3 + 120x_4 + 150x_5 + 200x_6 = 2{,}500{,}000$ Budget amount in \$

Computer solution and analysis.

Solution (Max)

Name	Final Value
Return	306910

Name	Final Value
x_1	100
x_2	4920
x_3	100
x_4	6500
x_5	6360
x_6	1250

Constraints

Name	Cell Value	Status	Slack
1	250000	Binding	0
2	500000	Binding	0
5	500000	Not Binding	250000
3	0	Binding	0
6	53220	Not Binding	43220
7	2500000	Binding	0

Sensitivity analysis

Name	Final Value	Reduced Gradient
x1	100	-0.580952726
x2	4920	0
x3	100	-0.557142712
x4	6500	0
x5	6360	0
x6	1250	0

Constraints

Name	Final Value	Dual Price
1	250000	0.039404762
2	500000	0.030059523
5	500000	0
3	0	-0.113095215
6	53220	0
7	2500000	0.112857142

Limits

Name	Value
return	306910

Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
x_1	100	100	306910	100	306910
x_2	4920	4920	306910	4920	306910
x_3	100	100	306910	100	306910
x_4	6500	100	211550	6500	306910
x_5	6360	6360	306910	6360	306910
x_6	1250	1250	306910	1250	306910

The solution is as follows:

Alternative	No. of shares
1	100
2	4920
3	100
4	6500
5	6360
6	1250

Maximum return is $306,910

The constraints 1,2,3 and 7 are completely used up. The dual prices are as follows:

Constraint	Dual price $
1	0.039404762
2	0.030059523
3	-0.113095215
7	0.112857142

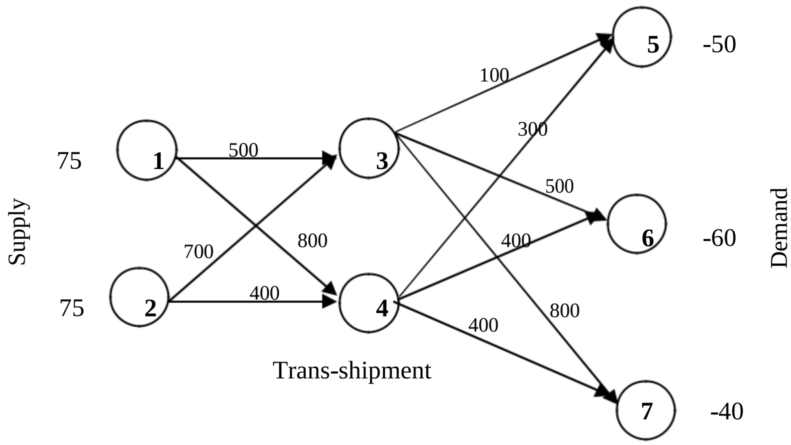
The slacks are in constraints 5 and 6 being $250,000 and $43,220 respectively

QUESTION 5

a) Degeneracy in linear programming is a case where one or more basic variables have values of zero in the optimal solution. It is dealt with by selecting another of the tied variables for removal resolving the cycling problem of degeneracy.

b)

i) The Network diagram is as follows



Arcs represent cost/revenue or capacities (cost of shelving in this case) Directed arcs show where flow is to.
Net flow=Flow into a node (-ve)+ flow out of a node (+ve)
Supply=demand. Nodes 1 and 2 are source nodes (with +ve net flow), nodes 3 and 4 are trans-shipment nodes (with zero net flow) and nodes 5,6,7 are sink node (with –ve net flows)

ii) The linear programming model for this problem is as follows
Taking x_{ij} to be units of shelves to be routed from supplies i to demand
j Minimize

$$z = 500x_{13} + 800x_{14} + 700x_{23} + 400x_{24} + 100x_{35} + 500x_{36}$$
$$+800x_{37} + 300x_{45} + 400x_{46} + 400x_{47}$$

Subject to

$x_{13} + x_{14} = 75$	node 1 (Apex)
$x_{23} + x_{24} = 75$	node 2 (Maxima)
$x_{13} + x_{23} - x_{35} - x_{36} - x_{37} = 0$	node 3 (Ujumi)
$x_{14} + x_{24} - x_{25} - x_{26} - x_{27} = 0$	node 4 (Pizza)
$-x_{35} - x_{45} = -50$	node 5 (Oceanic)
$-x_{36} - x_{46} = -60$	node 6 (Homegrown)
$-x_{37} - x_{47} = -40$	node 7 (Down town)

$x_{ij} \geq 0$

QUESTION 6

a) Let x_1 and x_2 be the number of shares invested in airline and insurance shares respectively. Then the objective function will be as follows:
Objective function (maximize)

$z = 2x_1 + 3x_2$ Share appreciation.

Subject to the following restrictions:
1. $40x_1 + 50x_2 \leq 100,000$ sh. Total investment.

2. $50x_1 \leq 40{,}000$ sh. Investment in insurance.
3. $x_1 + 1.5x_2 \geq 2600$ sh. Dividends.
4. $x_1, x_2 \geq 0$ Non-negativity of number.

b) Reduced cost represents the amount that objective function coefficient of a non-basic decision variable must improve (increase in this case) to be put into the basis. In this case since reduced cost is equal to zero for both variables, it means they are in the basis.

Dual prices on the other hand mean the amount that share appreciation will improve in case any of the limiting constraints is increased by one unit. This occurs for constraints that the slack is exactly zero. In this case total investment and investment in insurance do have positive dual prices while Dividends does have zero dual price (can not increase share appreciation if increased by one unit).

c) From the computer solution, the client's money should be invested as follows, to satisfy the restrictions: 1500 shares to be invested in Airline shares, and
800 shares to be invested in insurance shares, to give an optimum quarterly share appreciation of sh. 5400.

d) For the optimal decision to remain the airline shares appreciation should not be lower than 2.5, but can be any higher amount. That is, the optimum solution is insensitive to increase in the share appreciation.

QUESTION 7

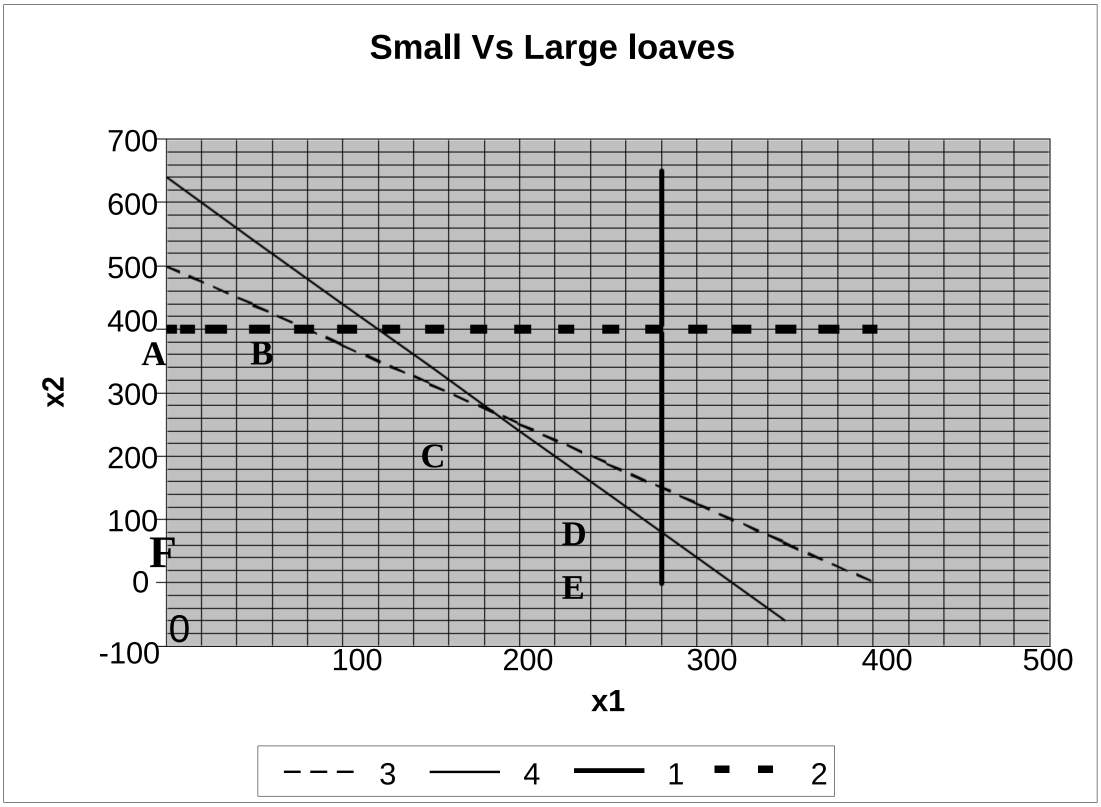
a) Formulation of the problem.
Take x_1 to be the number of large loaves and x_2 to be the number of small loaves. Objective function

$z = 5x_1 + 3x_2$ Profit

Constraints.

Line	1	$x_1 \leq 280$	Maximum number of large loaves.
Line	2	$x_2 \leq 400$	Maximum number of small loaves.
Line	3	$10x_1 + 8x_2 \leq 4000$	Space × (1000^{-3}) m^3
Line	4	$25x_1 + 12.5x_2 \leq 8000$	Hours × (1000^{-3})
	5	$x_1, x_2 \geq 0$	Non-negativity.

Using graphical method.



The feasible area is that enclosed by corner points ABCDEF

Corner points are where the optimum feasible solution exists. The upper points are to be considered for maximization problem.

At corner point A, Line 2 and $x_1=0$ intersect

So $x_2= 400$

Putting these value in the objective function gives the following

$z = 5x_1 + 3x_2=5\times0 + 3\times400=1200$ Profit$_A$

At corner point B, Line 2 and Line 3 intersect

$x_2= 400$ and $x_1=\frac{4000 - 8 \times 400}{10} = 80$

Putting these value in the objective function gives the following

$z = 5x_1 + 3x_2=5\times80 + 3\times400=1600$ Profit$_B$

At corner point C, Line 3 and Line 4 intersect

The equations for the lines are:

Line 3 $10x_1 + 8x_2 = 4000$ (1)

Line 4 $25x_1 + 12.5x_2 = 8000$ (2)

Multiplying equation (1) by 25 and equation (2) by 10 gives the following

$(10x_1 + 8x_2 = 4000) \times 25$

$(25x_1 + 12.5x_2 = 8000) \times 10$

$250x_1 + 200x_2 = 100{,}000$ (3)

$250x_1 + 125x_2 = 80{,}000$ (4)

Deducting equation (4) from Equation (3) gives:

$75x_2 =20{,}000$

$\Rightarrow x_2 =266.7$

$$\Rightarrow x_1 = \frac{4000 - 8 \times 266.7}{10} = 186.7$$

Putting these values in the objective function gives the following

$z = 5x_1 + 3x_2 = 5\times186.7 + 3\times266.7 = 1733$ $\quad$ $Profit_C$

At point D, Line 1 and Line 4 intersect.

$x_1 = 280$

$$\Rightarrow x_2 = \frac{8000 + 280 \times 25}{12.5} = 80$$

The profit at this point is then equal to:

$z = 5x_1 + 3x_2 = 5\times280 + 3\times80 = 1640$ $\quad$ $Profit_D$

At point E, Line 1 and $x_2 = 0$ intersect

$x_2 = 0 \Rightarrow \quad x_1 = 280$

$z = 5x_1 + 3x_2 = 5\times280 + 3\times0 = 1400$ $\quad$ $Profit_E$

Comparing these profits, it is at point C that profit is maximized.

So the solution is that:

$x_1 = 186$ $\quad$ No. of large loaves produced.

$x_2 = 266$ $\quad$ No. of small loaves produced.

And the maximum profit is $Profit_C$=Shs. 1,733

NOTE: Two methods can be used to solve the problem. It is easily solved using the graphical rather than the simplex method, since it is just two variables and sensitivity analysis is not required.

b) To solve this kind of problem (linear programming problem) the following procedure is followed:

- First, the problem has to be formulated. That is, the objective function and constraints are determined.
  Objective function is that which is to be optimised.
  Constraints are the limitations in resources.

- Secondly, the method of solving is determined. In this case, of a two-variable problem, the better method to use is graphical method, rather than simplex method.

- Thirdly, the constraints are taken as equalities and a line graph drawn. The unwanted regions are shaded out. Resulting region indicates the feasible region. The optimum point exists where there are corner points, which show extreme amounts. For maximization it is the outer ones to the right and up. For minimization it is the lower side.

- Lastly, the profit is determined at those points where there is maximum profit, is the point to be used.

NOTE: This part simply asks for the procedure followed.

QUESTION 8

a)

i) Simplex method will be appropriate.
Formulation of problem.
Objective function.
Let x_1 and x_2 be the number of Deluxe and Professional bicycle frames produced respectively per week.

$z = 1000x_1 + 1500x_2$ Profit sh.

Constraints:

$2x_1 + 4x_2 \leq 100$ Aluminum alloy
$3x_1 + 2x_2 \leq 80$ Steel alloy
$x_1, x_2 \geq 0$

In standard form:

$0 = z - 1000x_1 - 1500x_2 + 0s_1 + 0s_2$
$100 = 2x_1 + 4x_2 + s_1 + 0s_2$
$80 = 3x_1 + 2x_2 + 0s_1 + s_2$

Table 1

	x_1	x_2	s_1	s_2	Solution	Ratio	
s_1	2	4	1	0	100	25	⇐
s_2	3	2	0	1	80	40	
z	-1000	-1500	0	0	0		
	⇑						

Table 2

x_2	1/4	1	1/4	0	25	50	
s_2	2	0	-1/2	1	30	15	⇐
z	-250	0	375	0	37,500		
	⇑						

Table 3

x_2	0	1	3/8	-1/4	17.5	
x_1	1	0	-1/4	1/2	15	
z	0	0	312.5	125	41,250	

Stop here
The optimal weekly production schedule is as follows:
Deluxe bicycle Frame = 17.5 ≈17
Professional bicycle Frame = 15

ii) Let Δ_1 be the change in profit from Deluxe bicycle frame.
Δ_2 be the change in profit from Professional bicycle frame. So
$C_1 = 1000 + \Delta_1$ and $C_2 = 1500 + \Delta_2$ limit of profit.
From the final table:
To avoid entry of

s_1 $\Rightarrow 312.5 - 1/4\Delta_1 > 0$ $\Rightarrow$ $\Delta_1 < 1250$
s_2 $\Rightarrow 125 + 1/2\Delta_1 > 0$ $\Rightarrow$ $\Delta_1 > -250$

From the two conditions:
$-250 < \Delta_1 < 1250$ and
$750 < C_1 < 2250$

To avoid entry of

s_1 $\Rightarrow 312.5 + 3/8\Delta_2 > 0$ $\Rightarrow \Delta_2 > -833.33$
s_2 $\Rightarrow 125 - 1/4\Delta_2 > 0$ $\Rightarrow -\Delta_2 > -500 \Rightarrow \Delta_2 < 500$

So from the two conditions:
$-833.33 < \Delta_2 < 500$
And C_2 varies as follows
$666.7 < C_2 < 2000$

NOTE: This problem could be solved graphically with part (i) Easily determined. Part (ii) Limits will be determined from equating slopes of the objective function which has coefficients with constraints nearest to it.
For part (ii), accurate drawings will be required. Intuition will have to be followed and there will be an assumption that fractions are possible.

b) The technique is really involving.
Assumes fractions are possible, which is not really the case like here where we cannot make ½ a bicycle frame.

QUESTION 9

a)

i) A feasible solution is one that satisfies the objective function and given constraints
ii) Transportation problem is a special linear programming problem where there a number of sources and destinations and an optimum allocation plan is required. Total demand equal total supply
iii) Assignment problem is a special kind of transportation problem where the number of sources equals the number of destinations. That means for every demand there is one supply.

b) This is a case of assignment problem.
Assignment problems usually require that the number of sources equal the number of supply. Here there are 5 districts and only 4 salespersons. A dummy salesperson E is introduced with zero ratings.

		Districts				
		1	2	3	4	5
	A	92	90	94	91	83
Sales persons	B	84	88	96	82	81
	C	90	90	93	86	93
	D	78	94	89	84	88
	E	0	0	0	0	0

By following the Hungarian method:
Firstly:
For each row, the lowest rating is reduced from each rating in the particular row. This results to a row reduced rating table. Then all the zeroes are to be crossed by the least number of vertical and horizontal lines. If the number of lines equal the number of rows (or columns = 5 in this case) then the final assignment has been determined. Otherwise the following steps are followed.

	1	2	3	4	5
A	9	7	11	8	0
B	3	7	15	1	0
C	4	4	7	0	7
D	0	16	11	6	10
E	0	0	0	0	0

Secondly, for each column, the lowest rating is reduced from every rating in the particular column. In this case the table will remain the same since the dummy salesperson has ratings of zero for every district.
Thirdly a revision of the opportunity-rating table is done.
The smallest rating in the table not covered by the lines is taken (in this case it is one). This is reduced from all the uncrossed ratings and added to the ratings at the intersection of the crossings. Then all the zeroes are to be crossed by the least number of vertical and horizontal lines. If the number of lines equal the number of rows (or columns = 5 in this case) then the final assignment has been determined.

Otherwise the following steps are followed.

	1	2	3	4	5
A	8	6	10	8	0
B	2	6	14	0	0
C	4	4	7	0	8
D	0	16	11	6	11
E	0	0	0	0	1

Third step is repeated as follows:

	1	2	3	4	5
A	6	4	8	8	0
B	0	4	12	0	0
C	2	2	5	0	8
D	0	16	11	8	13
E	0	0	0	2	3

Still the optimal solution has not been reached. Third step is again repeated to give the following table:

	1	2	3	4	5
A	6	2	6	8	0
B	0	2	10	0	0
C	2	0	3	0	8
D	0	14	9	8	13
E	0	0	0	4	5

An optimal assignment can now be determined since the number of lines crossing the ratings is equal to 5.
Lastly, the assignment procedure is that a row or column with only one zero is identified and assigned. This row or column is now eliminated. The other zeroes are then assigned until the last zero is assigned. This step-by-step assignment is shown on the following table from the first one to the fifth one.

District

		1	2	3	4	5
	A	6	2	6	8	$\boxed{0}$ 4
Sales person	B	0	2	10	$\boxed{0}$ 5	0
	C	2	$\boxed{0}$ 3	3	0	8
	D	$\boxed{0}$ 1	14	9	8	13
	E	0	0	$\boxed{0}$ 2	4	5

The assignment is as follows

Salesperson	District	Rating
A	5	83
B	4	82
C	2	90
D	1	78
Total rating		333

The total rating is 333.

QUESTION 10

a) Sensitivity analysis measures how sensitive a linear programming solution is to changes in the values of parameters. These parameters include the coefficients of objective function, limiting resources and non-limiting resources.

So sensitivity analysis involves changing any of these parameters and showing how the linear programming problem is affected.
Dual values indicate the additional improvement of the solution due to additional unit of limiting resource. In that way, the additional improvement of solution is the price worth of paying to release a constraint

b) Let x_1, x_2 and x_3 be the units of desktop 386, Desktop 286 and laptop
486 Maximize profit
$Z=5000x_1+3400x_2+3000x_3$
Subject to

$x_1+x_2\leq500$ limit of desktop models
$x_3 \leq250$ limit of laptop model
$x_3 \leq120$ limit of 80386 chips
$x_2+x_3\leq400$ limit of 80286 chips
$5x_1+4x_2+3x_2\leq2000$ hours available
Assumptions $x_1, x_2, x_3 \geq 0$

- Linearity/proportionality
- Divisibility
- Deterministic
- Additive

c)

i) The optimum product mix is that the numbers of units to produce
are Desktop 386-120
Desktop 286-200
Laptop 486-200
Maximum profit is
$Z=5000\times120+3400\times200+3000\times200=$Sh1,880,000
Unused resources include the following
JK computers can still produce 180 more desktop models (500-120-200) and 50 laptop models (250-200)
For used up resources the prices to pay for any additional unit are as
follows Sh150 for 80386 chip
Sh90 for 80286 chip
Sh20 for any hour

ii) The range for the variables x_1, x_2 and x_3 are to indicate where the number of units can change without affecting the basic solution

The range for the constraints indicate the extent the resources can be changed without altering the basic solution of the linear programming problem

iii) The dual value of 80386 chip is Sh 150. That is the addition increase in profit due to increase of one chip. So if the company increases the number of chips by 10, the additional profit will be $10\times150=$Sh1,500.

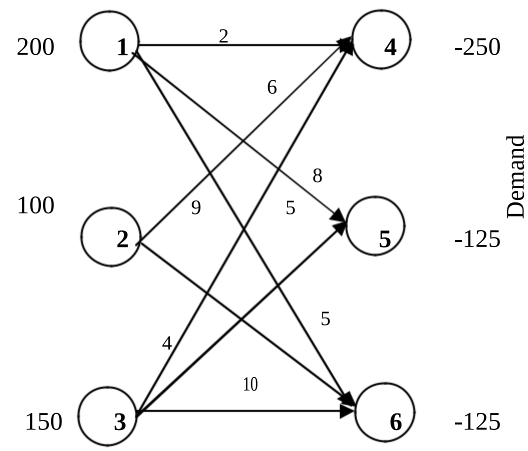
QUESTION 11

a)

i) A linear program is one that optimises a linear objective function with given decision variables subject to conditions that the constraints are linear and the variables are non-negative.
ii) The basic feasible solution is the set of variable values at a corner point of the feasible region (set of values for decision variables that satisfy both non-negativity and the structural constraints)
iii) Degeneracy is a case where one or more basic variables have values of zero in the optimal solution.
iv) Alternate optimal solution is said to exist when the number of optimal solutions is infinite and a given constraints is binding (further improvement on objective function is prohibited)
v) Redundant constraint is one that has no effect on the solution space. It contributes nothing to the solution space.
vi) Range of optimality represents the limits for which a solution remains optimal
vii) Artificial variable is one that is added to the LHS of starting basis so as to preclude yielding of infeasible (negative) solution for a mixed constraint situation.
viii) Reduced cost represents the reduction in profit due to addition of a non-limiting resource
ix) Dummy destination is one that is introduced so as to take care of excess supply. It has zero profit or cost.
x) Assignment problem is a special kind of problem where there are a number of supplies to satisfy equal number of destinations

b)

i)



Key

Node	
1	Thika
2	Naivasha
3	Machakos
4	Nairobi
5	Mombasa
6	Kisumu

ii) Minimize

$$z = 2x_{14} + 8x_{15} + 5x_{16} + 6x_{24} + 9x_{26} + 4x_{34} + 5x_{35} + 10x_{36}$$

Subject to

$x_{14} + x_{15} + x_{16} = 200$ node 1 (Thika)

$x_{24} + x_{26} = 100$ node 2 (Naivasha)

$x_{34} + x_{35} + x_{36} = 150$ node 3 (Machakos)

$- x_{14} + x_{24} - x_{34} = -250$ node 4 (Nairobi)

$- x_{15} - x_{35} = -125$ node 5 (Mombasa)

$- x_{16} + x_{26} - x_{36} = -125$ node 6 (Kisumu)

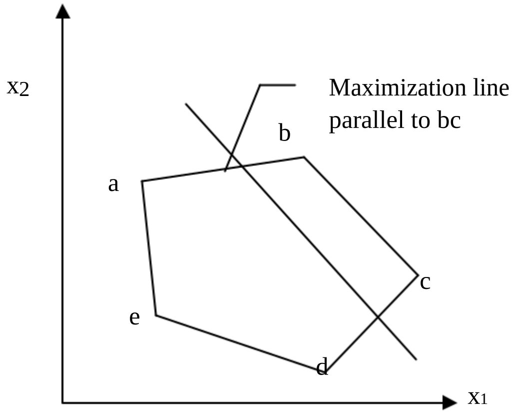
$x_{ij} \geq 0$ $i=1,2,3$

$j=4,5,6$

QUESTION 12

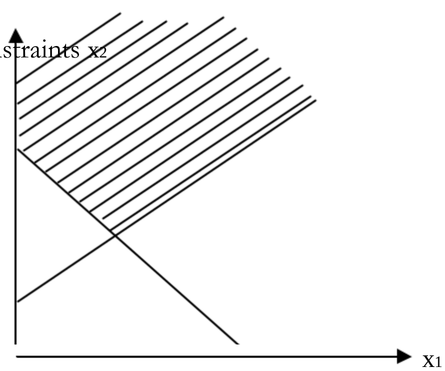
a)

i)

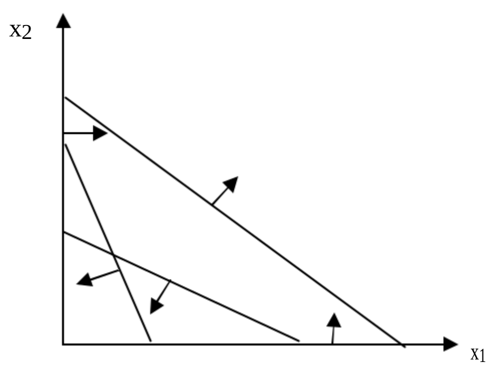


Alternate optimal solution exists when the number of optimal solution is infinite and the constraint parallel to objective function is binding.. More than one combination of values for the decision variables will result in the optimal value for the stated objective.

ii) Unbound solution is one that extends without limit. This reflects the nature of model constraints



iii) Infeasible solution exists where there is no point that satisfies the constraints and non-negativity conditions.



b)

i) Linear programming because it usually seeks to optimize a given function subject to various constraints. This is the case here.

ii) In formulation, the objective function is the one that is to be optimized. It has the following form

$Z=c_1x_1+c_2x_2+\ldots\ldots..c_nx_n$

Subject to the constraints:

$a_{11}x_1+a_{12}x_2+\ldots\ldots..a_{1n}x_n(\leq \geq =)\ b_1$

$a_{21}x_1+a_{22}x_2+\ldots\ldots.a_{2n}x_n(\leq \geq =)\ b_2$

$a_{m1}x_1+a_{m2}x_2+\ldots\ldots.a_{mn}x_n(\leq \geq =)\ b_m$

Where

c_i- Coefficient of decision variable i.

x_i- I_{th} decision variable (activity that competes for limited resources)

i. function to be optimise and forms the criteria

a_{ji}- Coefficient of j_{th} constraint for the I_{th} variable

b_j- Right hand side constant for the I_{th} constraint

m- Number of structural constraint

n- Number of decision variables

iii) Production technicians, measurement technicians, profit center personnel, cost center personnel. The required parameters will be reaction times, profit information, cost information and timing information.

QUESTION 13

a) Transportation problem involves movement of some homogenous commodity from various origins or sources of supply to a set of destinations each demanding specific levels of commodity. The main aim is to allocate supply from origins so as to optimize a criterion while satisfying the demand of each destination.
Assignment on the other hand is a special kind of transportation model where the numbers of sources of supply are equal to destinations

b)

i) Let x_1- the number of spot announcement on television

x_2- the number of spot announcement on radio

Maximize rating

$Z=600x_1+200x_2$

Subject to

$x_1+x_2\leq 30$ combined coverage

$x_2\leq 25$ maximum radio announcement

$x_2\geq x_1$ relation between radio and television announcement

$x_{1,2}\geq 0$

Solving the problem by simplex method as follows

Table 1

	x_1	x_2	s_1	s_2	s_3	s_4	Sol	b_i	
s_1	1	1	1	0	0	0	30	30	
s_2	0	1	0	1	0	0	25	∞	
s_3	1	-1	0	0	1	0	0	0	
s_4	1200	300	0	0	0	1	25500	21.25	⇐
=z	-600	-200	0	0	0	0	0		
	⇑								

Table 2

s_1	0	1.75	1	0	0	$-\frac{1}{1200}$	8.75	11.67	⇐
s_2	0	1	0	1	0	0	25	25	
s_3	0	-1.25	0	0	1	$-\frac{1}{1200}$	-21.25	17	
x_1	1	0.25	0	0	0	$\frac{1}{1200}$	21.25	85	
z	0	-50	0	0	0	$\frac{1}{2}$	12750		
		⇑							

Table 3

x_2	0	1	$4/3$	0	0	$-1/900$	$11\ 2/3$	
s_2	0	0	$-4/3$	1	0	$1/900$	$13\ 1/3$	
s_3	0	0	1.25	0	1	$-13/3600$	$-6\ 2/3$	
x_1	1	0	$-1/3$	0	0	$1/900$	$18\ 1/3$	
z	0	0	$200/3$	0	0	$4/9$	$13333\ 1/3$	

The iteration is stopped here since there is no more negative value in the z row.

x_1 should be $18\,{}^1\!/_3 \approx 18$ spot announcement on television

x_2 should be $11\,{}^2\!/_3 \approx 12$ spot announcement on radio

ii) Radio is likely to reach a large number of people even though it cannot really give the visual aspect which television has an advantage on.

iii) Letting Δ be a change on the coefficient of x_1 then from the final table,

$200/3 - 1/3\,\Delta > 0 \Rightarrow \Delta > 200$ also

$4/9 + 1/900\,\Delta > 0 \Rightarrow \Delta > -400$ so that Δ>200 for the basic solution obtained to change. In effect the rating required to change the number of television spot announcement is 600+200=800 and above. That is the rating should be above 800 for the number of television spots to increase.

iv) The restriction to relax are the s_1 and s_4.Combined coverage should be altered and the budget too because they do increase the total rating of advertisement (Combined coverage by $200/3$ for one unit relaxed and $4/9$ for every budget increase)

v) Any possible increase in advertising budget will increase total rating by only $4/9$, which is very small.

TOPIC 7

QUESTION 1

Optimal decision using:

a) Max-min criterion – Choose decision that maximizes the minimum profit. Min-max –choose decision that minimizes the maximum loss.

Decision alternatives		Worst outcome	
	D1	150	
	D2	140	
	D3	180	⇐Decision taken
	D4	160	

b) Max-max criterion – Choose decision that maximizes the maximum profit. Min-min –choose decision that minimizes the minimum loss.

Decision alternatives		Best outcome	
	D1	250	⇐Decision taken
	D2	225	
	D3	220	
	D4	230	

c) Min-max regret criterion –from regret table, choose the decision that minimizes the maximum regret. Regret = maximum payoff for a state of nature less the payoff of a given state in a decision alternative.

E.g. regret for: $D_1\theta_1 = 220 - 150 = 70$

$D_3\theta_1 = 210 - 190 = 20$

Regret table:

States of Nature

Decision alternative		θ_1	θ_2	θ_3	θ_4	θ_5	Max	Either
	D1	70	0	50	0	0	70	⇐Decision
	D2	40	85	30	50	25	85	
	D3	0	40	35	20	70	70	⇐Or this
	D4	30	15	0	10	90	90	

d) Maximum expected payoff –assuming equal likelihood of states of nature, decision that maximizes the expected payoff determined is taken.

For example:

Expected payoff for D_2 = Payoff $(D_2\theta_1 + D_2\theta_2 + D_2\theta_3 + D_2\theta_4 + D_2\theta_5)/5$

$= (180 + 140 + 200 + 160 + 225)/5 = 181$

Decision alternative		Expected Payoff	
	D1	203	⇐Decision taken
	D2	181	
	D3	194	
	D4	198	

QUESTION 2

a) Min-max

Decision alternatives		Worst outcome	
	D1	250	
	D2	225	
	D3	220	⇐Decision taken
	D4	230	

b) Min-min

Decision alternatives		Best outcome	
	D1	180	
	D2	140	⇐Decision taken
	D3	180	
	D4	160	

c) Min-max regret

Regret = loss of a given state in a decision alternative less minimum loss for a given state of nature.
E.g. regret for $D_3\theta_5$ = 180 - 160 = 20
Regret table:

States of Nature

Decision alternative		θ_1	θ_2	θ_3	θ_4	θ_5	Min	
	D1	0	85	0	50	90	90	
	D2	30	0	20	0	65	65	⇐Decision taken
	D3	70	45	15	30	20	70	
	D4	40	70	50	40	0	70	

d) Minimum expected loss

Decision alternative		Expected loss	
	D1	203	
	D2	181	⇐Decision taken
	D3	194	
	D4	198	

QUESTION 3

a) Expected payoff for a decision = Σ (Payoff; $\times$ f(θ) is
Where i = 1, 2, 3, 4 decision alternative
j = 1, 2, 3, 4, 5, 6 states of nature

Decision alternatives		Expected payoff	
	D1	342.4	
	D2	359.6	
	D3	330	
	D4	378.6	⇐Decision taken

b) Expected value under certainty: Under certainty given any state of nature a decision maker will choose the alternative with the highest payoff as follows:

	States of nature						
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	
Certain payoff	350	420	540	500	400	450	
Probability	0.18	0.1	0.16	0.24	0.2	0.12	Total
Expected value	63	42	86.4	120	80	54	**445.4**

c) Expected value of perfect information is equal to expected value under certainty less the expected value under uncertainty
Value in (b) –Value in (a) = 445.4 – 378.6 = 66.8

QUESTION 4

a) Let Di be the decision alternative, where i
(1) 0 stock 100 bond % of 1million family trust investment
(2) 10 stock 90 bond % of 1million family trust investment
(3) 20 stock 80 bond % of 1million family trust investment
□
□
□
(11) 100 stock 0 bond % of 1million family trust investment

θj – states of nature where
j – 1. Solid growth (12% bond: 20% stock)
2. Inflation (18% bond: 10% stock)
3. Stagnation (12% bond: 8% stock)

The payoffs in $ '000' are as follows:

	States of nature						
	θ_1	θ_2	θ_3	Max	Min	α-0.4	Equally
D_1	200	100	80	200	80	128	126.7
D_2	192	108	84	192	84	127.2	128
D_3	184	116	88	184	88	126.4	129.3
D_4	176	124	92	176	92	125.6	130.7
D_5	168	132	96	168	96	124.8	132
D_6	160	140	100	160	100	124	133.3
D_7	152	148	104	152	104	123.2	134.7
D_8	144	156	108	156	108	124.8	136
D_9	136	164	112	164	112	132.8	137.3
D_{10}	128	172	116	172	116	138.4	138.7
D_{11}	120	180	120	180	120	144	140

Payoff = (% bond x bond Yield + % stock x stock. Yield) × $1,000,000
E.g. $D_1\theta_2$ = (0 × 12% + 100% × 20%) 1,000,000 and
$D_7\theta_3$ = (60% × 12% + 40% × 8%)1,000,000
= (0.6 × 0.12 + 0.4 × 0.8)1,000,000
= $104,000

b) Max-max - D_1 with payoff of $200,000.
Max-min - $D_{11}$with payoff of $120,000.
Hurwicz - $D_{11}$with payoff of $144,000.
Equally likely - D_{11} with payoff of $140,000.

NOTE: Payoff in Hurwicz = (0.4 × Max payoff + 0.6 × min payoff) for a given decision alternative

E.g. D_3 = (0.4 × 184 + 0.6 × 88) $126,400

Regret table:

	States of nature			
	θ_1	θ_2	θ_3	Max
D_1	0	80	40	80
D_2	8	72	36	72
D_3	16	64	32	64
D_4	24	56	28	56
D_5	32	48	24	48
D_6	40	40	20	40
D_7	48	32	16	48
D_8	56	24	12	56
D_9	64	16	4	64
D_{10}	72	8	4	72
D_{11}	80	0	0	80

Either D_1 or D_{11} with a regret of $80,000 will be taken

NOTE: Regret = maximum payoff for a given state of nature less the payoff of a given state in a decision alternative

E.g. $D_7\theta_2$ = 180 - 148 = 32

c)

	States of nature		
	θ_1	θ_2	θ_3
Probability	0.4	0.25	0.35

	Expected value $'000'	
D_1	133	
D_2	133.2	
D_3	133.4	
D_4	133.6	
D_5	133.8	
D_6	134	
D_7	134.2	
D_8	134.4	
D_9	134.6	
D_{10}	134.8	
D_{11}	135	⇐Best strategy

Expected value for decision = Payoff in θ_1 × 0.4 + Payoff in θ_2 × 0.25 + Payoff in θ_3 × 0.35 E.g. D_1 = 200 × 0.4 + 100 × 0.25 + 80 × 0.35 = 133

d) With perfect information

	States of nature			
	θ_1	θ_2	θ_3	Total
Strategy under states of nature	200	180	120	167,000

Here the maximum payoff for any state of chosen total expected payoff:

= 200 × 0.4 + 180 × 0.25 + 120 × 0.35

=167,000

QUESTION 5

a) Payoff = (Revenue / Household × No. of households) – Initial cost Payoffs in millions

Plan	Revenue	No. of households 10,000	20,000	30,000	40,000	50,000	60,000
I	150	-5.5	-4	-2.5	-1	0.5	2
II	180	-5.2	-3.4	-1.6	0.2	2	3.8
III	200	-5	-3	-1	1	3	5
IV	240	-4.6	-2.2	0.2	2.6	5	7.4

b) Optimistic approach means that the max-max criterion is used.

Plan	Max	
I	2	
II	3.8	
III	5	
IV	7.4	⇐Adopt Plan IV

Min-max regret means, from the opportunity loss table, the minimum of the maximum is actually chosen.
The opportunity loss table.
Opportunity loss or regret = max payoff for a given number of households less the payoff of a given number of household and given plan. E.g. Plan III for 40,000 household, = 0.26 - 1 = 1.6 million shillings

Plan	No. of households 10,000	20,000	30,000	40,000	50,000	60,000	Max	
I	0.9	1.8	2.7	3.6	4.5	5.4	5.4	
II	0.6	1.2	1.8	2.4	3	3.6	3.6	
III	0.4	0.8	1.2	1.6	2	2.4	2.4	
IV	0	0	0	0	0	0	0	⇐Adopt

c) Given the probabilities, the payoff table will change to be as follows
Payoff = Payoff as determined from part of a) multiplied by the given probability under the respective pricing plans.

e.g. for Plan II for 3,000 household = -1.6 × 0.2
= -0.32

Expected payoff = sum of all the payoffs for a given plan.

Plan	No. of households 10,000	20,000	30,000	40,000	50,000	60,000	Expected	
I	0	-0.2	-0.125	-0.4	0.15	0.4	-0.175	
II	-0.26	-0.34	-0.32	0.06	0.4	0.57	0.11	⇐Adopt
III	-0.5	-0.6	-0.2	0.2	0.6	0.5	0	
IV	-0.92	-0.55	0.05	0.39	0.5	0.37	-0.16	

The pricing plan to follow is Plan II, which gives a higher expected payoff of sh110, 000.

d) The approach used in part (b) is that of decision making under uncertainty. Probabilities of occurrence as much as the outcomes are not known with certainty.
The approach in (c) on the other hand is decision making under risk. Probabilities of occurrence of an event is known with given amount. This gives expected payoff for any decision undertaken.

QUESTION 6

a) Decision making under risk is when decisions are made using already known probabilities for states of nature or outcomes. The probabilities can come from previous data.

Decision making under certainty is when a decision is made where there is no prior probabilities for states of nature or outcomes.

b) Decision tree is a diagrammatic representation of decisions given different states of nature. Nodes and branches are used to represent the decisions and outcomes from given decisions.

Probability tree is a diagrammatic representation of the sequence of outcomes given certain probabilities.

c) Minimax criterion-involves choosing the alternative with minimum regret from choice of maximum regrets from given events.
Maximax criterion- involves choosing the alternative with maximum payoff from choice of maximum payoffs from given events.

d) Pure strategy in a game is where each player knows exactly what the other player is going to do. The same rule is still followed each time.
Mixed strategy is where there is a combination of the rules followed. Each player does not know what the other player is going to do. In this case probabilities are used to find what each player will do. The main aim is to maximize expected gains or to minimize losses.

e) Games represent a competitive situation where players aim to gain from each other.
Games with more than two persons represent real life situation where there are more than two persons as players. Each person seeks to gain from the others.
Non-zero sum games represent situation where it is not necessarily that what one losses is gained by another.

QUESTION 7

a)

i) Dominance is a principle where superior strategies of a player are said to dominate his inferior ones. This is because there is no incentive to use inferior strategies
ii) Saddle point is said to exist when the maximum of row minimum coincides with the minimum of the column maxima in a payoff matrix
iii) Mixed strategy is when players in a game use a combination of strategies and each player is always kept guessing as to which course of action is to be selected by the other player at a particular occasion. Players do not play the same strategy each time
iv) Value of a game is the payoff of play when all the players of the game follow their optimal strategies.

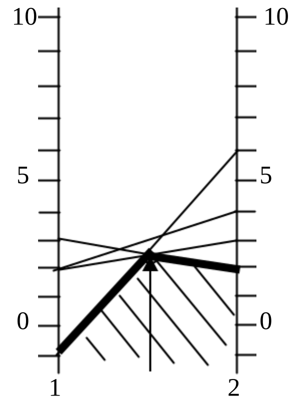
b)

i) The payoff table is as follows

		Player B strategies				Row minimum	
		1	2	3	4		
Player A Strategies	1	2	2	3	-1	-1	
	2	4	3	2	6	2	⇐
Column maximum		4	3	3	6		
			⇑	⇑			

It is not possible to determine the value of the game because player A's stategy does not result to player B's strategy. There is no saddle point .In this case a mixed strategy is adopted to determine the value of the game.

ii)



The value of the game is determined as the highest point of the shaded area (shown on the drawing by an arrow)

This represents the highest loss on average that player A can have given player B's winnings. This point can be determined by extracting a 2×2 matrix made up of the lines that intersect to make the point. In this case the matrix is as follows.

		Player B	
		y	1-y
Player A	x	3	-1
	1-x	2	6

The expected value of the game E(x,y)=3xy-(1-y)x+2y(1-x)+6(1-x)(1-y)

Differentiating partially with respect to x and y and equating to zero gives the following.

$$\frac{\partial E(x, y)}{\partial x} = 3y - (1 - y) - 2y - 6(1 - x) = 0 \Rightarrow -7 + 8y = 0 \Rightarrow y = \frac{7}{8}$$

$$\frac{\partial E(x, y)}{\partial y} = 3x + x + 2(1 - x) - 6(1 - x) = 0 \Rightarrow -4 + 8x = 0 \Rightarrow y = \frac{1}{2}$$

So the optimal mixed strategy for the game is

Player A $\quad x = \frac{1}{2},\ 1 - x = \frac{1}{2}$

Player B $\quad y = \frac{7}{8},\ 1 - y = \frac{1}{8}$

Value of the game is obtained by substituting the mixed strategy in the expression for expected value of the game as follows.

$$E(x, y) = 3 \times \frac{1}{2} \times \frac{7}{8} - 1 \times \frac{1}{2} \times \frac{1}{8} + 2 \times \frac{7}{8} \times \frac{1}{2} + 6 \times \frac{1}{2} \times \frac{1}{8} = \frac{21}{16} - \frac{1}{16} + \frac{14}{16} + \frac{6}{16} = \frac{40}{16} = 2.5$$

Notes

The player with only two strategies available is drawn first represented by the two lines 1 and 2 (in this case player A).

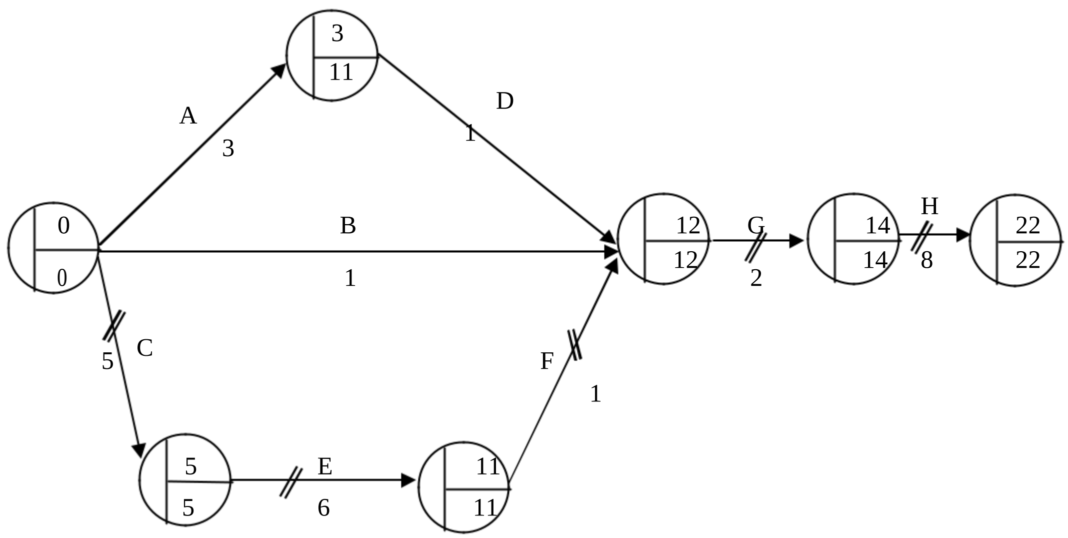
The two lines 1 and 2 are divided into equal number of expected winnings of player B as shown.

Player B's strategy can then be drawn. This is drawn by a straight line joining the winning of Player B given player A's strategy. For example for player B's first strategy, a line is drawn from 2 (if player A plays strategy 1) to 4 (if player A plays strategy 2). This is done for every strategy of player B.

TOPIC 8

QUESTION 1

a) Final network diagram is as follows.



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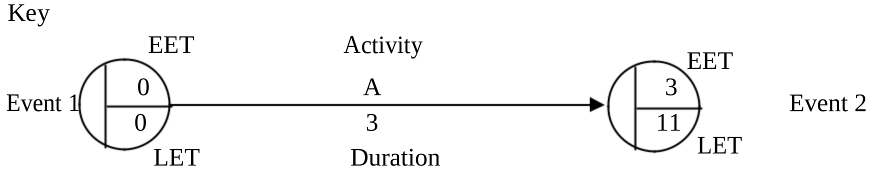


EET-Earliest event time (from forward pass)
LET-Latest event time (determined from backward pass)

b) Slack being the delay that an activity can have without delaying the overall time for the project is calculated as follows.
The slack for activity A is determined as follows: Slack s_A=LET_2-EET_1-d_A =11-0-3=8.

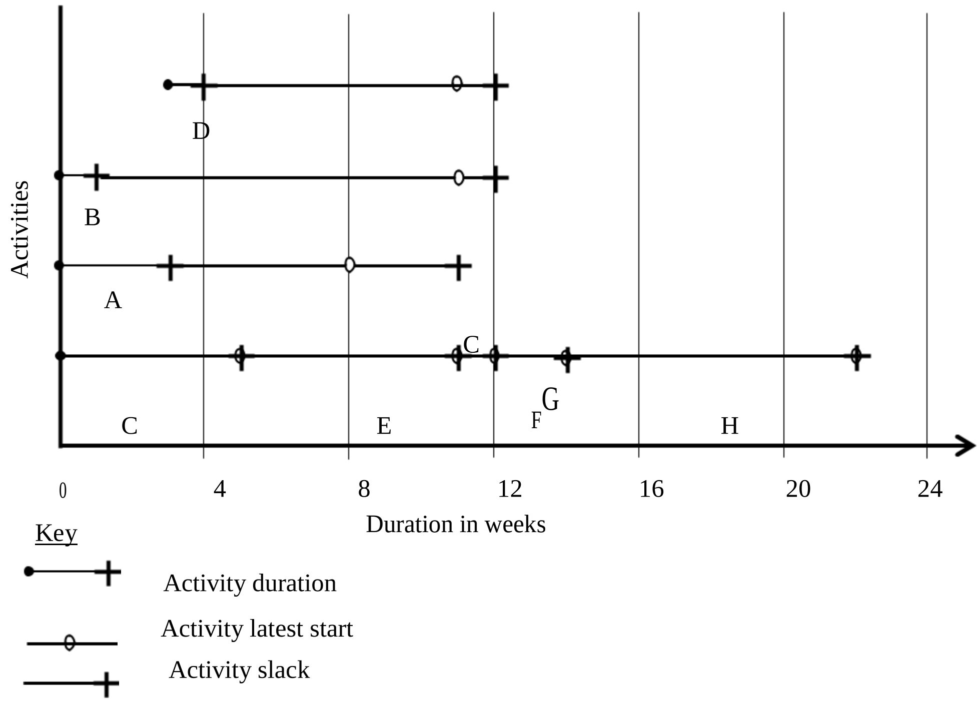
Where: LET_2 - LET of end event
EET_1 - EET of start event
d_A - Duration of activity A

LET of the end event less the EET of the start event less the duration of the activity.
This is done for all activities as follows:

Activity	LET_2	EET_1	Duration D	Slack = LET_2-EET_1-D
A	11	0	3	8
B	12	0	1	11
C	5	0	5	0
D	12	3	1	8
E	11	5	6	0
F	12	11	1	0
G	14	12	2	0
H	22	14	8	0

c) Critical path is formed of activities that are having zero slack. Delaying any of these activities will lead to delay of completion of the project. From the table, the activities having zero slack are C-E-F-G-H. These activities have been marked on the diagram with –//–

d) The time chart is drawn as follows:



The scheduling flexibilities can be clearly seen from the time chart showing the slack, duration and the latest start for the activities

Activities CEFG and H cannot be delayed at all since they form the critical path

Activity A can start as late as 8th week
Activity B can start as late as 11th week
Activity D can start as late as 11th week
} Without affecting project completion.

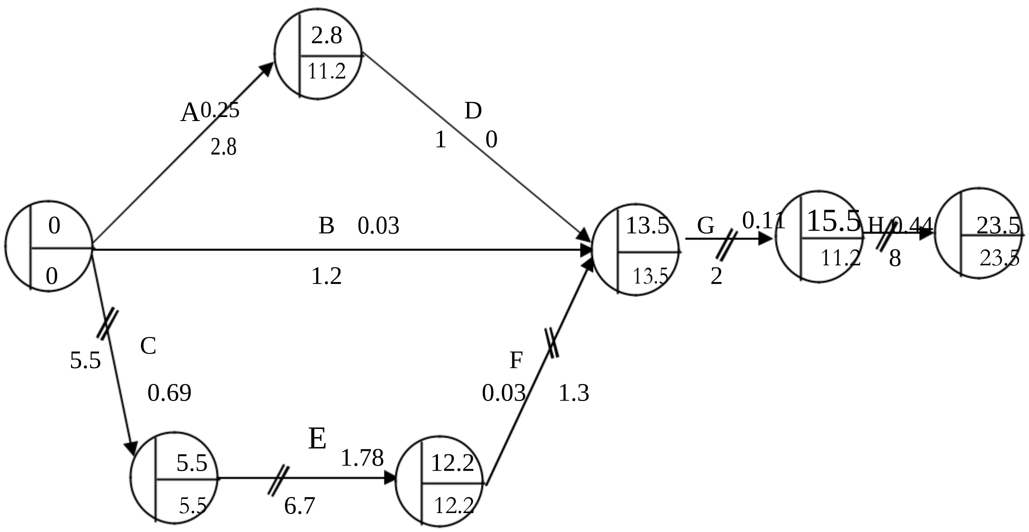
QUESTION 2

a) Calculation of estimated duration d_{ij} and standard deviation of duration σ_{ij} from the data of time estimates for the various activities is as follows:

$$d_{ij} = \frac{a_{ij} + 4m_{ij} + b_{ij}}{6} \quad \text{and } \sigma_{ij}^2 = \left(\frac{b_{ij} - a_{ij}}{6}\right)^2$$

Where: a_{ij}- optimistic time
b_{ij}- pessimistic time
m_{ij}- most likely time

Activity	a_{ij}	m_{ij}	b_{ij}	d_{ij}	σ_{ij}^2	Slack	Comment
A	1	3	4	2.8	0.25	8.4	Not critical
B	1	1	2	1.2	0.03	12.3	Not critical
C	4	5	9	5.5	0.69	0	Critical
D	1	1	1	1.0	0.00	9.7	Not critical
E	4	6	12	6.7	1.78	0	Critical
F	1	1	2	1.2	0.03	0	Critical
G	1	2	3	2.0	0.11	0	Critical
H	6	8	10	8.0	0.44	0	Critical



b) The slacks in this situation are all more than in the situation where optimistic/pessimistic times are not included.

c) The critical path remained the same being C-E-F-G-H.

d)

i) The variance for the whole project is as follows

$$\sigma^2=\sigma_A^2+\sigma_B^2+\sigma_C^2+\sigma_D^2+\sigma_E^2+\sigma_F^2+\sigma_G^2+\sigma_H^2$$

$$\sigma_2=0.25+0.03+0.69+0+1.78+0.03+0.11+0.44$$

$$\sigma_2=3.6$$

The expected time of completion is T=23.5 weeks. The probability of completion of project within t=22 weeks is as follows:

$$P(t \le T)=P\ z \le \frac{(t-T)}{\sigma}$$

$$=P\ z \le \frac{(22-23.5)}{3.3}$$

$$= P\ (z \le -0.826)$$

From normal distribution table at z=-0.79, the required probability is (0.5-0.2967)=0.2033 So the probability of completing the project in 22 weeks is 0.2033.

ii) Expected time of completion is T = 23.5 weeks. So the probability of finishing the project within the earliest expected completion date is

$$P(t \le 23.5) = P\ z \le \frac{23.5-23.5}{\sqrt{3.3}} = P(z \le 0)$$

From normal distribution tables at z=0 the probability =0.5. So the probability of finishing the project within the earliest expected completion date is 50%

iii) The probability of the project taking more than 30 days to complete

$$P(t \geq 30) = P\ z \leq \frac{30 - 23.5}{\sqrt{3.3}} = P(z \leq 3.26)$$

From normal distribution tables at z=3.56 the probability =0. So the probability of the project being completed after 30 weeks =0.

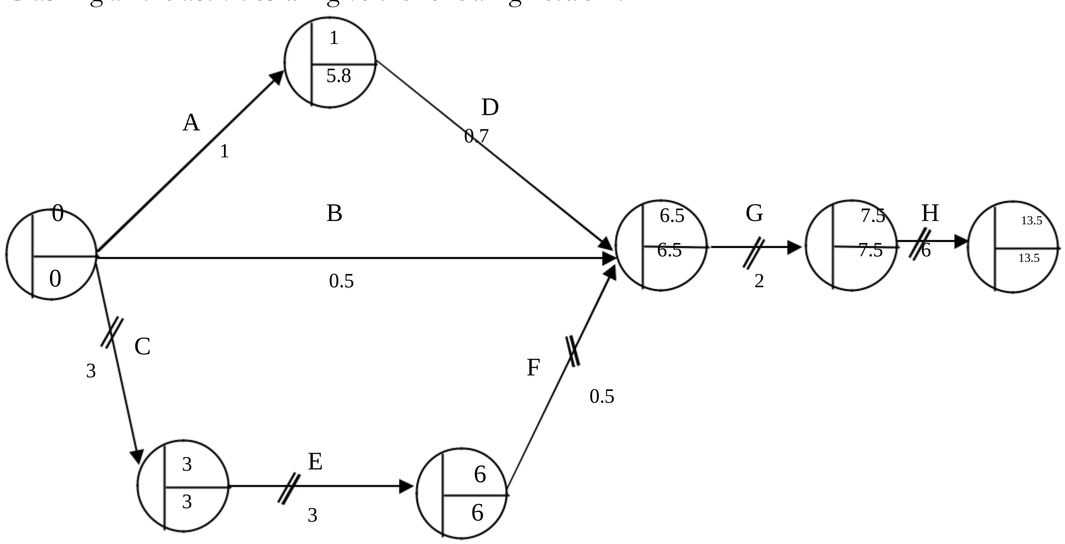
QUESTION 3

Linear slope R for the activities is determined as

$$R = \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

Activity	R $/week
A	3.25
B	1.6
C	4.5
D	3.3
E	10
F	1.6
G	5
H	25

Crashing all the activities will give the following network.



The critical path is still C-E-F-G- H with completion time being 13.5 weeks

The schedule of cost is as follows

Comment	Time in weeks	Direct cost (000)	Indirect cost (000)	Opportunity cost (000)	Total cost (000)
Normal	22.0	140.9	27	9	177.9
Crash F	21.5	141.7	26.5	7	177.2
Compress C	20.5	146.2	25.5	5	178.7
Crash C	19.5	150.7	24.5	3	180.2
Crash G	18.5	155.7	23.5	1	182.2
Compress E	17.5	165.7	22.5	0	190.2
Compress E	16.5	175.7	21.5	0	197.2
Crash E	15.5	185.7	20.5	0	206.2
Compress H	14.5	210.7	19.5	0	230.2
Crash H	13.5	235.7	18.5	0	254.2

The minimum cost occurs at 21.5 weeks indicated on the table.

Notes:
The linear slope R indicates the crash cost per week crashed. If any activity is to be crashed, the one with lowest crash cost per week and in the critical path is chosen first until it is exhausted.
To find the minimum cost schedule, a table is drawn as shown. Direct cost is made up of normal cost and any crash cost. Indirect cost will have the fixed and variable cost while opportunity cost will be there for any period exceeding 17 weeks.
Normal time cost is made up of:

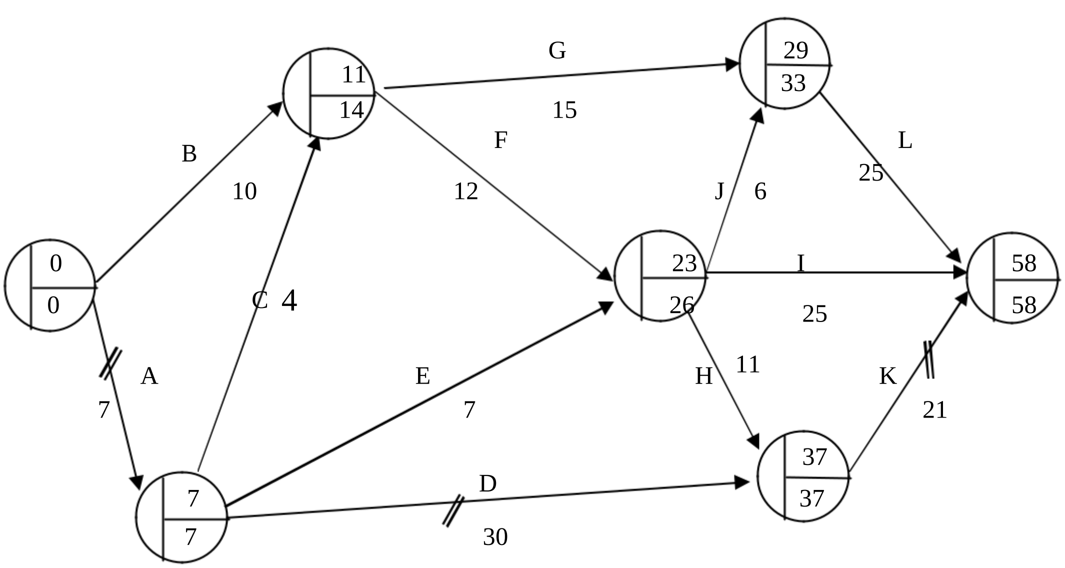
- Direct cost of $140,900 determined by addition of all direct the costs of activities.
- Indirect cost is equal to $5,000 + elapsed time × 1000 = $27,000
- Opportunity cost=(elapsed time-17) weeks×2000=(22-17)×2000=$10,000

Total normal cost =140,000+27,000+10,000=$177,900
With the aim of reducing costs and time, activities are crashed and compressed depending on which activity is in the critical path and has the lowest crash cost per week at that time. This is done until the project crash time is reached.

So to start, activity F is crashed because it is in the critical path and has the lowest crash cost per week. This adds onto direct cost the amount of $1,600×0.5 weeks and reduces indirect cost by $1,000×0.5 weeks and opportunity costs by $2,000×0.5 to give the total cost of $177,200.
The next activity in the critical path with lowest crash cost per week is C. This is first compressed by one week before crashing it. This process continues with costs determined on the table. Care has to be taken as to the maximum possible crashing of activity.
Activities not in the critical path are not crashed because they add cost without reducing time.

QUESTION 4

a) To get the project completion time and critical path the network is drawn as follows.



Doing a forward pass on the activities, the earliest event times are determined as shown on the network diagram. The completion time is determined to be 58 hours.
On doing a backward pass the latest event times are determined. From these event times the critical path can be identified as those that are between events where both earliest and latest event times are equal. From the network diagram the critical path marked with

Critical path is A - D - K.

b) Activities G and E are not part of critical path. Activity G's earliest start is 11 hours of which activity E can start too since its latest start is 16 hours (LET_2 - d_E = 23 - 7 = 16). So the activities G and E can be performed at the same time without delaying the project.

c) One person can perform activities A, G and I without delaying the overall completion of the project. This is because activities G and I are not part of critical path. G can start after activity A since its earliest start time is 11 hours which is well over the latest end time for activity A. Activity I can start after G and still be completed without delaying project time.

d) Activities G and L are not in the critical path. Activity G can be delayed for 7 hours and activity L can be delayed for 4 hours without delaying the project time.

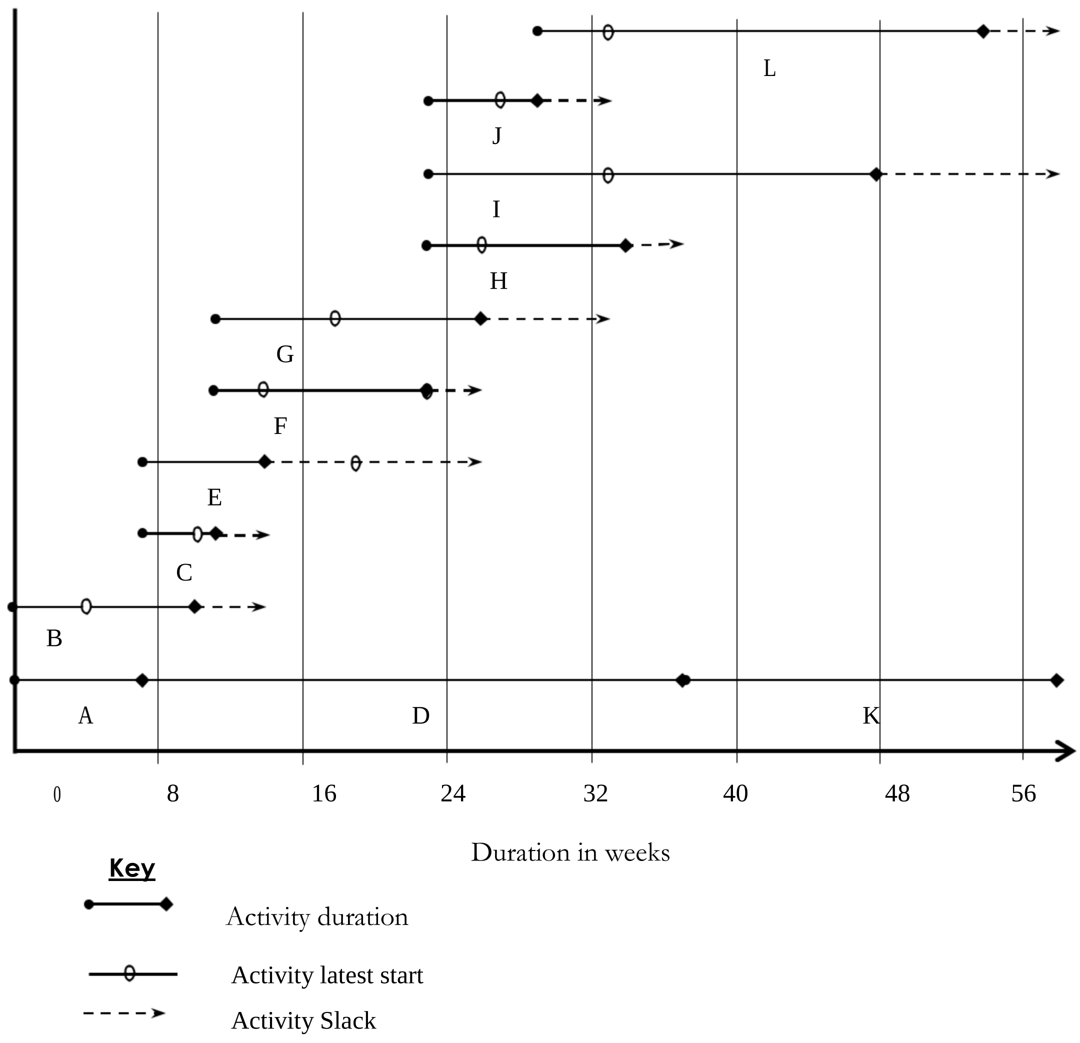
e) Delaying activity G by 3 hours does not make the activity critical. Furthermore, delaying the activity L by 4 hours just makes this activity critical. Therefore the overall effect is that there is no delay in the project.

NOTES:
This question could be answered using a table of scheduled times and a Gantt chart.
The table of activity times is as follows.

Activity	EET(n)	LET(n+ 1)	Duration D	Slack= LET(n+ 1)-EET(n)-D	Comment
A	0	7	7	0	critical
B	0	14	10	4	not critical
C	7	14	4	3	not critical
D	7	37	30	0	critical
E	7	26	7	12	not critical
F	11	26	12	3	not critical
G	11	33	15	7	not critical
H	23	37	11	3	not critical
I	23	58	25	10	not critical
J	23	33	6	4	not critical
K	37	58	21	0	critical
L	29	58	25	4	not critical

N Start Node; n+1 - end node of activity

To be able to see the scheduling flexibilities, a Gantt chart can be drawn as follows.



QUESTION 5

a)

i) Network planning is the arrangement of activities and events in a diagram to create a logical relationship from start to end making up a project.

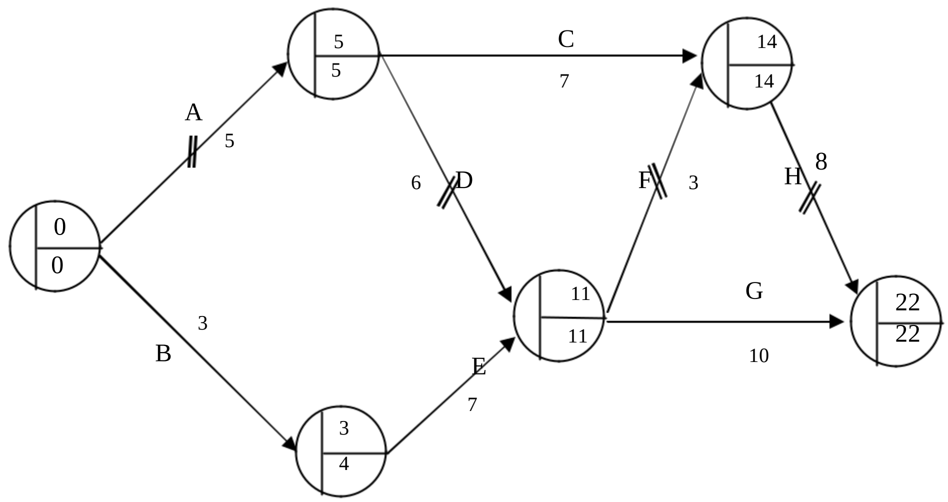
ii) Activities are those actions that take up resources and time.

iii) Events indicate start or completion of activities

iv) Critical path is a combination of activities that, if any of them is delayed, then the projects duration will also delay. That is, they are critical to the projects duration. They are activities with zero floats.

v) Float is the amount of time an activity can be delayed without delaying the overall time of the project.

b) The activity on node network can be changed to event on node for ease of seeing the critical path. This also reduces the number of nodes encountered.



i) The project will take 22 weeks to complete.
ii) Activity D cannot be delayed without delaying the entire project, since it is an activity in the critical path A-D-F-H.
iii) Activity E can start as early as after 3 weeks and can be delayed for one week without delaying the project time.

NOTES:
The question could be approached in a different way by not changing the activity on node diagram. Instead a chart of float and arrangement of activities will be used.

A chart of duration and float is as follows
From the on node network the preceding activities can be read from the network shown by arrows entering the node (activity).

Activity	*Preceding Activities*
A	-
B	-
C	A
D	A
E	B
F	D,F
G	D,E
H	C,F

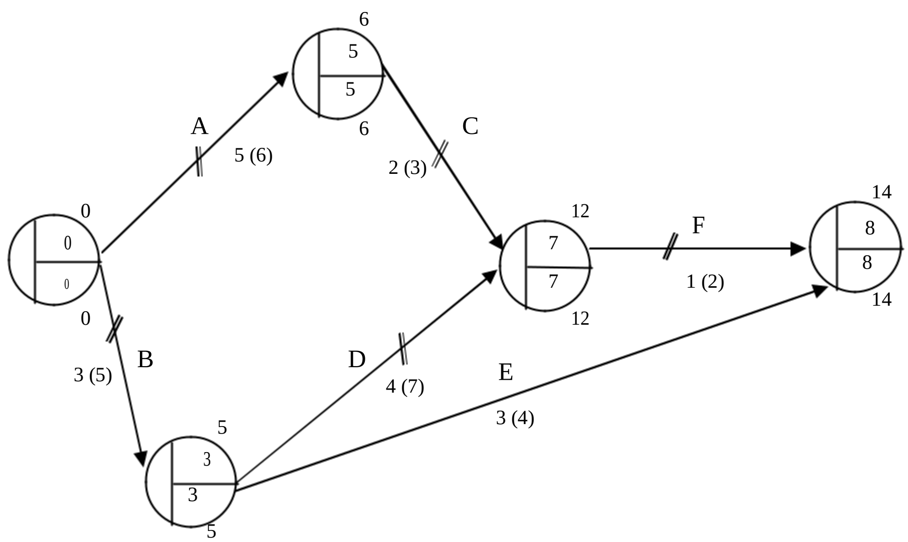
Comment	**Activity**	**Activity**	**Duration**	**EET(n)**	**LET(n+ 1)**	**Float**
Start	A		5	0	5	0
Start	B		3	0	4	1
	C	A	7	5	14	2
	D	A	6	5	11	0
	E	B	7	3	11	1
	F	E,D	3	11	14	0
End	G	E,D	10	11	22	1
End	H	F,C	8	14	22	0

n - start node n+1 - annode of given activity

QUESTION 6

a)

i) The shortest time to finish the project is determined by crashing all the activities. The network diagram drawn using crash times is drawn as follows. (Note the normal duration for activities is in brackets and event times are above and below the events)



From the network diagram, the critical paths are A-C-F and B-D-F.
The project crash duration is 8 days.
The crash cost for the project is (70+50+15+55+30+17.5)× 1,000=KSh. 237,500

ii) The cost schedule table is as follows.

Normal	14	9	0	179
Compress D	13	4.5	5	179.5
Compress D	12	0	10	180
Crash D	11	0	15	185
Crash F (A-C-F & B-D-F critical)	10	0	22.5	192.5
Crash C	10	0	27.5	197.5
Compress B	9	0	37.5	207.5
Crash B	9	0	47.5	217.5
Crash A	8	0	57.5	227.5

The linear crashing cost per day $R = \dfrac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$

Activity	R
A	10
B	10
C	5
D	5
E	10
F	7.5

It is economical to do the project within the normal duration of 14 days without crashing any activity.

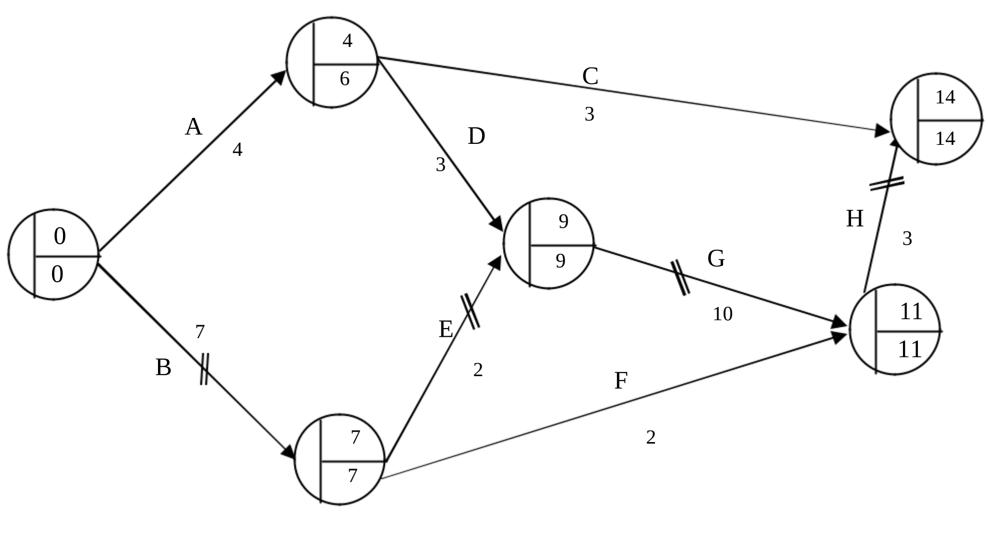
Notes:
When normal activity durations are used, there is only one critical path B-D-F. So the activity with the lowest crash cost per day is D. It is compressed first by one day then again compressed and finally crashed. The opportunity cost decreased to zero while the additional crash cost increased progressively by Sh 5,000. At this point there are two critical paths A-C-F and B-D-F. Activity F is chosen to be crashed although it has a higher cost per day than activity C. This is because to crash C, activity B has to be compressed to be able to reduce the project duration by one day.
Activity C can then be crashed although this does not reduce the duration because of the other critical path B-D-F.
Compressing activity B after C is crashed results to reduction in project time of one day.
Crashing activity B will require that activity A is crashed too to come to the minimum project duration of 8 days.
At this point crashing terminates.

b) The different approaches to displaying project information are Gantt chart, project evaluation and review technique PERT, critical path analysis CPA and resource schedule charts.
Gantt chart involves displaying the activities on a graph against time. A line shows start, duration, end and float of activity.
CPA involves displaying project activities on network. The logical relationship between activities is shown together with activity durations. From this network, the critical activity can be determined.
PERT involves displaying project activities on a network like in CPA. The times for the duration used here are uncertain. So the expected time is used instead.
Resource schedule chart involves presenting project activity resources required and what is available on chart. For every resource, in a project, a resource chart is drawn.

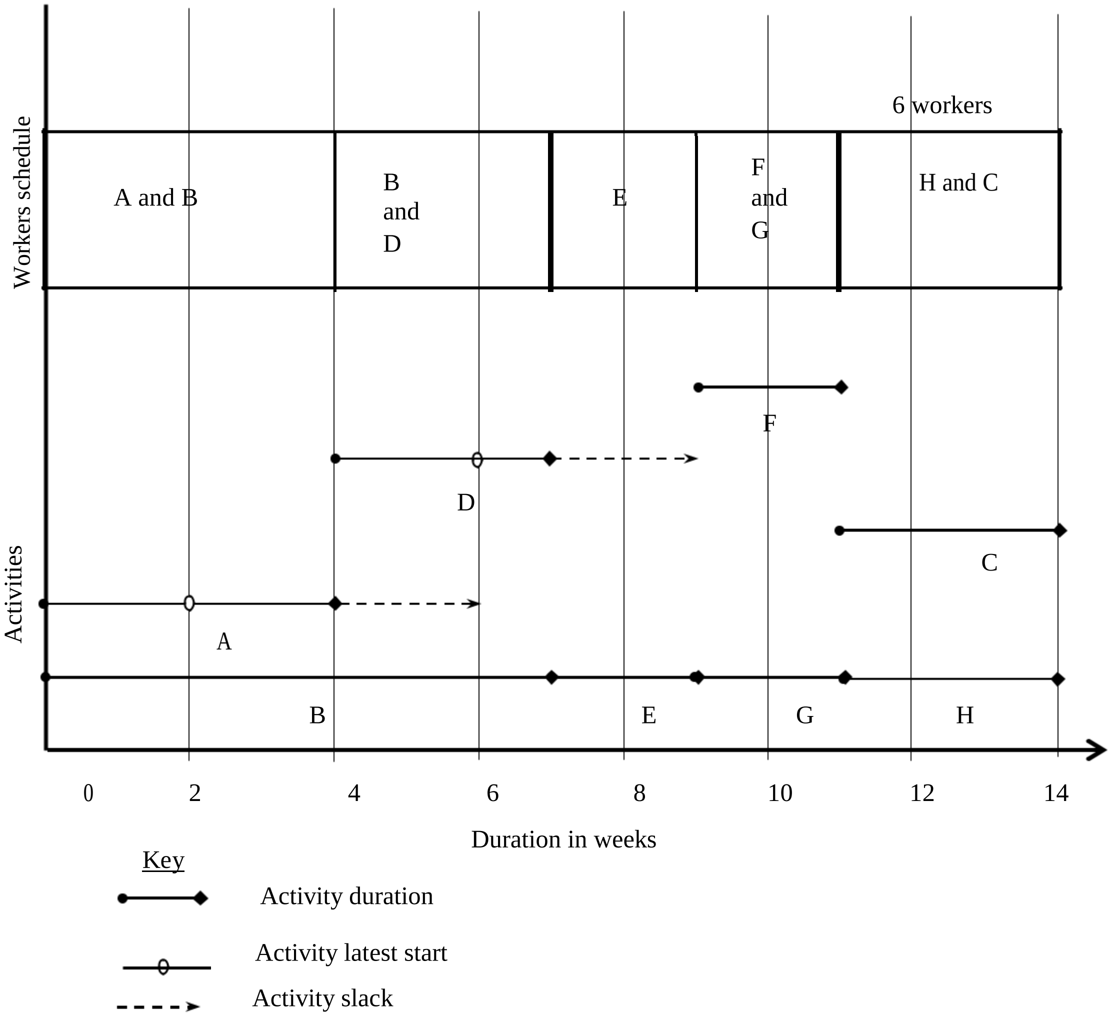
QUESTION 7

a) The four attributes that make the Beta distribution be chosen as representing distribution of times for PERT analysis are:
- It is uni-modal
- It has finite limits
- Can assume flexible shapes
- There is goodness of estimates of expected duration and variance

b)
i)



ii) Critical path is B-E-G-H with shortest project duration being 14 weeks.
iii) The workers schedule merged with the Gantt chart for the project is as follows:



iv) Lillian Wambugu can engage six workers through out the duration of the project and she will finish the project within the 14 weeks . Notes:
Notice how activity C and F have been pushed to their ends of slack times to ensure a smooth resource engagement.

Part III: Comprehensive Mock Examinations

QUESTIONS - MOCKS

PAPER 1

Time allowed: 3 hours

Answer any THREE questions in SECTION I and TWO questions in SECTION II. Marks allocated to each question are shown at the end of the question. Show all your workings.

SECTION I

QUESTION ONE

a) Write down short notes on:
   i) Final demand. (2marks)
   ii) Technical coefficients. (2marks)
   iii) Closed model versus open model. (3marks)

b) Three industries, packaging P, Bakery B, and Flour F are related with the intermediate demand matrix below.

$$\begin{array}{ll} & \text{Output industry} \\ & \begin{array}{ccc} P & B & F \end{array} \\ \begin{array}{r} P \\ \text{Input Industry } B \\ F \end{array} & \begin{array}{ccc} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{array} \end{array}$$

The final demand for P, B and F are:

$$D = \begin{pmatrix} 20 \\ 20 \\ 50 \end{pmatrix}$$

Required:
   i) Determine the total output for products of industries P, B and F. (10 marks)
   ii) Comment on the total output for industries. (3 marks)

QUESTION TWO

a) Define the following terms as used in measures of dispersion:
   i) Platykurtic versus leptokurtic and mesokurtic. (6marks)
   ii) Coefficient of skewness. (2marks)
   iii) Quartile deviation. (2marks)

b) A survey by Marketing Society here in Kenya found out the following buying habits of household air conditioners considered a luxury.

Gross income per month	Number of consumers who buy Air conditioner
Up to 5,000	2
5,001 – 10,000	15
10,001 – 15,000	25
15,001 – 20,000	28
20,001 – 25,000	33
25,001 – 30,000	35
30,001 – 35,000	38
35,001 and above.	40

Required:

i) Calculate the arithmetic mean, median and mode. (5 marks)

ii) Does the survey show that the luxury item is consumed by high-income earners? Show your answer by calculating appropriate measure of skewness. (5 marks)

QUESTION THREE

a) In connection with probability, define the following terms

i) Compound event (2 marks)

ii) Mutually exclusive events (2 marks)

iii) Collective exhaustive events (2 marks)

iv) Equally likely events (2 marks)

v) Conditional probability (2 marks)

b) Given the following distribution of wages for 500 technicians in motor companies in industrial area.

Wages in KSh	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000
No. of technicians	30	40	250	100	60	20

A technician is chosen from the above group. What is the probability that his wages are?

i) Under KSh 5,000 (4 marks)

ii) Above KSh 6,000 (4 marks)

iii) Between KSh 5,000 and KSh 6,000 (2 marks)

QUESTION FOUR

a) Define the goodness of fit test. How is it applied in accounting? (5 marks)

b) A research studying the role of stress and its implication on personal life in respect of job change over by low cadre staff, came up with the following data. It relates to 30 firms over 3-year period

No. of people changing jobs in a year	0	1	2	3	4	5	6	7	8	9
Observed frequency	8	18	19	20	16	12	8	4	3	2

By fitting a Poisson distribution to get expected frequency, test its goodness of fit. (15 marks)

QUESTION FIVE

a) With reference to linear regression define the following terms:

i) Scatter diagram.

ii) Bivariate distribution.

iii) Positive correlation.

iv) Confidence interval.

v) Auto correlation. (10 marks)

b) The following data relates business turnover and staff of fast moving consumer goods company EAI Ltd:

Year	1993	1994	1995	1996	1997	1998	1999	2000
Business turnover in Millions of shillings	45	50	60	75	80	110	150	170
Staff	2,600	3,000	3,100	3,530	3,850	4,300	5,870	7,150

Required:

i) Fit an appropriate regression equation. (8 marks)

ii) Estimate the staff requirement when business turnover reaches Sh.200 Million. (2 marks)

SECTION II

QUESTION SIX

a) What are the underlying assumptions in linear programming? (5 marks)

b) Zadock ltd. manufactures various kinds of furniture namely ordinary chairs, baby cots, and executive chairs. All the products use raw material (wood), machine time (lathe) and manual labour (technicians). The requirements for the furniture are summarised as below.

	wood in Cubic meters	Lathe time in hours	Technician time in hours
Ordinary chair	10	3	2
Baby cots	12	5	3
Executive chairs	20	6	5

The estimated profits for each kind of furniture are Sh. 26, Sh. 35 and Sh. 50 respectively. There is a limitation of capacity, in that only 300 cubic meters of wood, 118 hours of lathe time and 90 hours of technician time are available.

This problem was solved using a spreadsheet and the following was obtained.

Target Cell (Max)

Name	Final Value
Z	856
X1	6
X2	20
X3	0

Constraints

Name	Cell Value	Status	Slack
wood	300	Binding	0
lathe	118	Binding	0
technician	72	Not Binding	18
X1	6	Not Binding	6
X2	20	Not Binding	20
X3	0	Binding	0

Variables

Name	Final Value	Reduced Gradient
X1	6	0
X2	20	0
X3	0	-2

Constraints

Name	Final Value	Dual Price
wood	300	1.78571428
lathe	118	2.71428571
technician	72	0

Coefficients

Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
X1	6	-3.5972E-14	700	6	856
X2	20	0	156	20	856
X3	0	0	856	0	856

Required:

i) Formulate the linear programming problem into standard form for input to the computer spreadsheet (5 marks)

ii) What are the units of each kind of furniture to be produced to maximise profits. What is the maximum profit? (5 marks) iii) Is there any resource that is not used up? For the used up resource if any, how much would you pay for one additional unit. (5 marks)

QUESTION SEVEN

a) Define the following as used in game theory:

i) N – person game. (1mark)

ii) Saddle point. (2marks)

iii) Strategy. (2marks)

iv) Rules of dominance. (3marks)

b) Peter and Ngomongo are second hand dealers of electronic equipment and shoes along Baricho road. Due to hard economic times and restriction by City Council, sales have been decreasing. Each of them came up with strategies to expand into the other business (they have monopoly now). Each of them knows what the other is considering thus influences each others decision. Peter had the following matrix of profit per day.

		Ngomongo: Expand	Ngomongo: Don't expand
Peter	Expand	1000	0
	Don't expand	-500	500

Required:

i) Interpret the matrix. (4marks)

ii) Solve the game to determine the average winning (loss) each cousin would have. (8marks)

QUESTION EIGHT

Vick Press Ltd is planning to expanding into offering security systems in people's homes scared of security. The following are activities and associated costs for the expansion:

		Normal		Crash	
Activity	Predecessor	Time weeks	Cost *Sh.*	Time weeks	Cost *Sh.*
A	-	10	10,000	7	12,000
B	A	35	50,000	33	52,000
C	A	4	7,000	3	8,000
D	C	25	20,000	25	26,000
E	B,D	5	5,000	4	4,500
F	C	2	4,000	2	4,000
G	F	4	30,000	4	30,000
H	G	2	15,000	1	25,000
I	E,H,L	1	4,000	1	4,000
J	-	8	12,000	4	24,000
K	-	12	24,000	10	20,000
L	J,K	4	6,000	2	2,000
M	J	8	10,000	6	8,000

Required:

i) Determine the critical path. (8marks)

ii) Determine the minimum time and minimum cost for the networks. (7marks)

iii) Given that, for every delay beyond 40th week, there is share of Sh.1,000 per week loss of profit. Is it advisable to crash the project from 51 to 45 weeks? Why? (5marks)

PAPER 2

Time Allowed: 3 hours

Answer any THREE questions in SECTION ONE and TWO questions in SECTION II. Marks allocated to each question are shown at the end of the question. Show all your workings.

SECTION I

QUESTION ONE

a) Kiko Manufacturing Ltd. wants to take a decision of introducing a new soap Kikope. The cost for introducing Kikope (Initial advertising, promotion and fixed cost for one year of production) is estimated at Ksh.30,000. The variable cost per bar of soap is Ksh.30 and the expected selling price is Ksh.50.

Required:
Draw the cost, revenue and profit functions and from it:
i) Determine the break-even level of production. (3 marks)
ii) Determine the profit on the sale of 2,500 soap bars. (2 marks)

b) Caterpillar Company Ltd. produces generators and drilling engines. The revenue function that describes total revenue for sales in a particular quarter of these engines is given by:

$R = 8x + 5y + 2xy - x^2 - 2y^2 + 20$
Where:
x – number of generator engines in thousands.
y – number of drilling engines in thousands.

Required:
At what quantities of x and y is revenue maximised in a particular quarter. (5 marks)

c) Define the terms systematic, stratified, multistage, cluster and quota sampling as used in statistical inference.

QUESTION TWO

a) Define the following:
i) Markov process. (2 marks)
ii) Cyclic chain. (2 marks)
iii) Absorbing state. (2 marks)
iv) State transition matrix. (2 marks)
v) Steady state. (2 marks)

b) In brand switching between different toothpastes named Closed-up CU, Cols-gate CG and Aquas-fresh AF, the state transition matrix below is obtained for a particular month.

		From		
		CU	CG	AF
To	CU	0 .5	0 .45	0 .1
	CG	0 .2	0 .7	0 .1
	AF	0 .1	0 .2	0 .7

Required:
i) If the market share for the different toothpastes were 0.4, 0.3 and 0.3 for CU, CG and AF for January 2002, what will be the market share in February 2002 and March 2002? (6marks)

ii) What is the market share equilibrium situation? (4marks)

QUESTION THREE

a) Write short notes on the following:
   i) Price relatives. (2marks)
   ii) Fixed base method. (2marks)
   iii) Chain base method. (2marks)
   iv) Weighted index number. (2marks)
   v) Quantity index number. (2marks)

b) The price index for 2001 with base 1985=100 for the listed goods is.

Goods	Price Index
X	100
Y	130
Z	180

Find the Arithmetic mean and geometric mean of these three indices. (5marks)

c) In a base period, salaries comprise 50% of selling price of product, materials and overheads 40%, and profits 10%. If salary structure rose by 20% and materials and overhead costs rose by 10% from base period. What must have been the percentage increase in selling price if profits remained 10% of selling price? (5marks)

QUESTION FOUR

a) What is a Venn diagram? How can it be used to determine probability of a given event? (4 marks)

b) Zinco International Corporation had 1,500 employees. In the year 2001, 300 employees got salary increment, 100 were promoted and 50 got both increment and promotion.

Required:
   i) How many employees got only a promotion? What is that probability? (2 marks)
   ii) How many employees got neither increment nor promotion? Give that probability (2 marks)

c)
   i) Explain the meaning of a random variable (2 marks)
   ii) Zebro Match Manufacturers finds that 5% of the matches in a box are defective. Determine the probability that out of a box containing 50 match sticks:
      a) None will be defective (4 marks)
      b) If the guarantee is not more than 4 will be defective, what is the probability that the box will meet guarantee quality? (6 marks)

QUESTION FIVE

a) Forecasting is the attempt to predict the future by using qualitative or quantitative means. Discuss the qualitative and quantitative techniques available for managers in predicting the future. (6marks)

b)
   i) What are the steps in time-series decomposition? (3marks)
   ii) Given the following data of production of vehicles by a local car assembly KMA Ltd. Determine the deseasonalised data and using the forecasting relationship forecast the production in May 2003.

Production in '000'

Year	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
2001	7.92	7.81	7.91	7.03	7.25	7.17	5.01	3.9	6.64	7.03	6.88	6.14
2002	4.86	4.48	5.26	5.48	6.42	6.82	4.98	2.45	4.51	6.38	7.55	7.59

(11marks)

SECTION II

QUESTION SIX

a) Define a dual problem in relation to the primal problem. (3 marks)

b) A fruit/juice kiosk along Lusaka road, VegFresh kiosk has come up with the following formulae of making natural juice for its industrial area customers.

- 5 litres lemonade: 2 dozen lemon, 2 kg of sugar, 2 ounces of citric acid and water
- 5 litres grapefruit: 11/2 kg grape fruit, 11/2 kg of sugar, 1 ounces of citric acid and water
- 5 litres orangeade: 11/2 dozen of oranges, 11/2 kg of sugar, 1 ounces of citric acid and water

The selling prices of the fruit juices per every 5 litre are:

Lemonade:	Sh. 37.50
Grapefruit:	Sh. 40.00
Orangeade:	Sh. 42.50

During the cold weather season (May 2002) VegFresh kiosk had in stock 2,500 dozen lemons,2,000 kg grape fruit, 750 dozen oranges, 5,000 kg of sugar and 3,000 ounces of citric acid

Required:

i) Formulate the linear programming problem (5 marks)

ii) Formulate the dual of this primal (5 marks)

iii) The computer solution of the dual was determined to be as follows (Minimise)

Name	Final Value
turnover	83958.3333

Name	Final Value
Lemons	0
Grapefruit	7.91666667
Oranges	15.8333333
Sugar	0
Citric acid	18.75

Constraints

Name	Cell Value	Status	Slack
lemonade	37.5	Binding	0
grapefruit	40	Binding	0
orangeade	42.5	Binding	0
Lemons	0	Binding	0
Grapefruit	7.91666667	Not Binding	7.916667
Oranges	15.8333333	Not Binding	15.83333
Sugar	0	Binding	0
Citric acid	18.75	Not Binding	18.75

Sensitivity Report

Name	Final Value	Reduced Gradient
Lemons	0	1999.99976
Grapefruit	7.91666667	0
Oranges	15.8333333	0
Sugar	0	1750
Citric acid	18.75	0

Constraints

Name	Final Value	Dual Price
lemonade	37.5	250
grapefruit	40	1333.33333
orangeade	42.5	500

Limits Report

	Target Name	Value		
	turnover	83958.3333		
	Adjustable Name	**Value**	**Lower Limit**	**Target Result**
	Y1	0	0	83958.33
	Y2	7.91666667	7.916667	83958.33
	Y3	15.8333333	15.83333	83958.33
	Y4	0	0	83958.33
	Y5	18.75	18.75	83958.33

What are the manufacturing quantities in May 2002 that maximises the turnover? And what is the maximum turnover? (4 marks)

iv) Which raw materials are not used up? What are the amounts? (3 marks)

QUESTION SEVEN

a) What is Laplace criterion? How does it differ from Hurwicz criterion? (4marks)

b) Sikoka health club specializes in provision of sports/exercise and medical/dietary advice to clients. The service is provided on residential basis and clients reside for whatever number of days that suit their needs. Budget estimates for the year ending 30 June 2003 is as follows:

1) Maximum capacity of center = 50 clients per day for 350 days in the year.
2) Clients will be invoiced at a fee per day. The budgeted occupancy level will vary with the client fee level per day and is estimated at different percentages of maximum capacity as follows:

Client fee per day in Sh.	Occupancy level	Occupancy % of maximum capacity
3,600	High	90
4,000	Most likely	75
4,400	Low	60

3) Variable costs are estimated at one of the three levels

High	1,900
Most likely	1,700
Low	1,400

4) The range cost levels reflect only the possible effect of the prulus price of goods and services

Required:

i) Summary of budgeted contribution to be earned by Sikoka Health Club for the year ended 30 June 2003 for each of the nine possible outcomes. (10marks)

ii) State the client fee strategy that will result from the use of the maximax, maximin and minimax regret decision criteria. (6marks)

QUESTION EIGHT

MMK Ltd. plans to conduct a survey. The following table shows the tasks involved, the immediately preceding tasks and for each activity duration the most likely estimate (L), the optimistic estimate (O) and the pessimistic estimate (P).

Activity	Preceding activity	Number of days Most likely (L)	Optimistic (O)	Pessimistic (P)
A	-	6	4	8
B	-	24	20	40
C	A	10	8	24
D	B	8	4	12
E	D	6	6	6
F	B	8	6	10
G	C,E,F	20	16	36
H	G	6	4	8
I	G	4	4	4
J	H	10	8	12
K	I,J	8	4	24

Using the project evaluation and review technique (PERT) the mean time M and standard deviation σ, for the duration of each task are estimated from the most likely (L), optimistic (O), pessimistic (P) estimates by using the formulae

$M=0.08333(4L+O+P)$

$\sigma=0.08333(P-O)$

Required:

a) Compute the mean duration and standard deviation for each task (9marks)

b) The project is budgeted to cost Sh.500,000. Actual costs per day are Sh.10,000. By first identifying the critical path from drawing a network diagram can the project be implemented within the budget. (9 marks)

c) What is the probability of finishing the project 4 days earlier than the expected duration? (2 marks)

PAPER 3

Time Allowed: 3 hours

Answer any THREE questions in SECTION I and TWO questions in SECTION II. Marks allocated to each question are shown at the end of the question. Show all your workings.

SECTION I

QUESTION ONE

a) How does differential calculus assist managers in their optimization problems when faced with single variables? (4marks)

b) The total cost function and demand function of Migoya Construction company (MCC) are as follows:

$C = x^2 + 16x + 39$ Cost in million shillings.
$P = x^2 - 24x + 117$ Price per building in million shillings.
x – number of buildings

Required:

i) Write down the expression of average cost per unit and graph it for x values between x=0 and x=8. (3marks)

ii) Write the expression for total revenue. (2marks)

iii) If total revenue is to be maximized, what is the price to be charged? (3marks)

iv) Elasticity of demand = $\frac{p}{x} \times \frac{1}{dp/dx}$

Determine the elasticity of demand for the quantity that maximizes total revenue. (5marks)

v) What is the price that maximizes profit? (3marks)

QUESTION TWO

a) A Markov process describes the movement among the different states of a system as a function of time. Give the various steps in using a Markov process for determining a state at (t + 2) and steady state two periods after given state. (6 marks)

b) A major bank BKR Ltd calculates the credit ratings of its credit card customers on a monthly basis. The ratings are poor, good and excellent depending on the payment history. The following matrix shows how the customers change from one category to the other in one month.

		To		
		Poor	Good	Excellent
From	Poor	0.8	0.18	0.02
	Good	0.2	0.75	0.05
	Excellent	0	0.16	0.84

Required:

i) Interpret the elements 0.84, 0.2, 0.18 and 0.16. (4 marks)

ii) Given that in August 2003, from customer base of 100,000 the accounts were classified as

Poor	30,000
Good	50,000
Excellent	20,000

What is expected in October? (10 marks)

QUESTION THREE

a) Write short notes on the following:
   i) Uniform distribution
   ii) Poisson distribution
   iii) Normal distribution
   iv) Beta distribution
   v) t-distribution (10 marks)

b) A performance test was done before and after training on 12 persons appointed in clerical positions in a cement factory warehouse in Athi River. Marks were awarded on a 10-point scale as follows,

Employee	A	B	C	D	E	F	G	H	I	J	K	L
Before training	4	5	3	7	8	6	5	9	10	6	4	3
After training	5	4	6	8	7	5	9	9	10	6	5	4

Did the training improve performance? (10 marks)

QUESTION FOUR

a) A sample should reduce as much as possible elements of bias since they deprive statistical result of its representativeness. Define a bias and describe at least **four** examples of ways that bias is introduced in a sample. (6 marks)

b) A researcher finds that 65% of a random sample of 100 technicians can be classified as highly dissatisfied with their job. With the scale used to measure job satisfaction, the percentage of highly dissatisfied people in all types of occupation has averaged 45%. Can the researcher conclude that a larger proportion of technicians is dissatisfied than is true for all occupations. (Use 5%level of significance) (4 marks)

c) In testing records generated by two sub departments in a large firm, the following were the incorrect records obtained.

	Number in sample	Mean	Standard deviation
Production	10	45	7
Sales	5	55	6

Is the difference in number of incorrect records significant? (10 marks)

QUESTION FIVE

a) What is the estimated standard error of a regression equation? (2marks)

b) A land resale firm, Shamba Nyingi Limited, sells land around the environs of Nairobi. The size and price data for a given year are as shown.

Size ('000' of square meters)	Price in Sh. Millions
2.3	4.2
2.4	4.5
3.0	4.8
2.6	4.9
2.6	5.3
3.0	6.2
2.7	7.5
3.6	8.5
2.0	3.5

Required:
   i) Determine the linear-regression equation of price on size. (8marks)
   ii) Describe the relationship between size and price. (2marks)
   iii) What do the equation coefficients represent? (3marks)

iv) What is the estimated standard error of the regression equation? (2marks)
v) Determine the prize when the size of land is 2,150 square meters. (3marks)

SECTION II

QUESTION SIX

a)
- i) What is sensitivity analysis on the objective function coefficients? Differentiate this with sensitivity analysis on the right hand side (RHS) constants (4 marks)
- ii) Differentiate linear programming from assignment and transportation problems. (4 marks)

b) In a linear programming problem the objective function and constraints were determined to be as follows.

Maximise $Z= -x_1-x_2+3x_3-2x_4$ Profit

Subject to : $x_1 + 3x_2 - x_3 + 2x_4 \leq 7$ Resource A

$-x_1 - 2x_2 - 4x_4 \leq 12$ Resource B

$-x_1 + 4x_2 + 3x_3 + 8x_4 \leq 10$ Resource C

Where $x_j \geq 0$, j=1,2,3,4

Required:
- i) By simplex method find the final table. (7 marks)
- ii) Give the optimal solution and the value of the corresponding objective function (2 marks)
- iii) For the variable x_2, give an interval for their objective function coefficient such that the present basic solution remains optimal. (3 marks)

QUESTION SEVEN

a) The marketing department of EAA Ltd developed a sales forecasting function for its washing powder and those of competitor Hankol Ltd. EAA Ltd has 3 strategies and Hankol Ltd has 4 strategies. The increase and decrease in quarterly sales revenue for the different combinations of strategies of EAA Ltd and Hankol Ltd are as shown in the payoff matrix:

		Hankol Ltd			
		1	2	3	4
	1	5,000	-2,000	12,000	-5,000
EAA Ltd	2	6,000	2,000	7,000	6,000
	3	- 2,000	0	- 400	7,000

Required:
Using graphical or otherwise determine the strategy available for EAA to pursue. (5marks)

b) A milk processing plant Oldonyo Kesses Ltd in Rift-Valley desires to determine how many kilograms of butter to produce per day to meet demand. Past records indicate the following patterns of demand:

Quantity in Kgs	Number of days which demand occurred
15	4
20	16
25	20
30	80
35	40
40	30
45	10

Stock levels are restricted to the range 15 – 45 kilograms (in multiples of 5 kg.) and butter left is disposed by giving the employees working in the factory at the end of the day. Cost of production is Sh.44 and selling price is Sh.50 per Kg.

Required:

i) Construct a payoff table. (10marks)

ii) Determine the production alternative that maximises expected profit. (5marks)

QUESTION EIGHT

a) In relation to project analysis define the terms:

- i) Latest start time. (1mark)
- ii) Earliest start time. (1mark)
- iii) Resource leveling. (3marks)

b) A project has 11 activities with the activities preceding constraints, time estimates and work team requirements as shown:

Activity	Preceding activity	Days	Work team
A	-	10	5
B	A	8	3
C	A	5	5
D	B	6	4
E	D	8	1
F	C	7	2
G	E,F	4	2
H	F	2	1
I	F	3	3
J	H,I	3	1
K	J,G	2	5

Required:

i) Prepare a network and indicate critical path. (6marks)

ii) Prepare a time chart and work team requirement. (6marks)

iii) Can the work team resource be levelled further? Why? (3marks)

ANSWERS - MOCKS

SUGGESTED ANSWERS TO MOCK EXAMS
SOLUTIONS FOR PAPER 1

QUESTION 1

a)

i) Final demand is the amount of production that is consumed by those outside the interconnected industry.
It is an additional production for a closed input-output system.

ii) Technical coefficients are the fractions of total units produced by industries that are consumed by interrelated industries. They form the internal demand to make up the input-output matrix.

iii) Closed model is the input-output model that have entire production being consumed by those participating in production.

Open model on the other hand is one in which some of the production is consumed by external bodies. There is external demand in this case.

b)

i) The Leontief open model is Mx+d=x
So rearranging the equation (I-M)x=d x=(I-M)$^{-1}$d
Where M-matrix of technical coefficients x-required production
d-the external demand
I-Identity matrix

Given that $M = \begin{pmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{pmatrix}$

$d = \begin{pmatrix} 20 \\ 20 \\ 50 \end{pmatrix}$ Then

$$(I-M) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.4 \end{pmatrix}$$

$$(I-M)^{-1} = \frac{1}{\text{Deter min ant}(I-M)} \times \text{Adjo int}(I-M)$$

Determinant (I-M) $= |I-M| = 0.5(0.16-0.04) - 0.1(0.08-0.02) + 0.1(0.04-0.04)$
= 0.06-0.006+0=0.054

Ad joint (I-M)=Transpose of the co-factors of (I-M)

$$\text{Co-factors of } (I\text{-}M) = \begin{pmatrix} 0.12 & -0.06 & 0 \\ -0.02 & 0.19 & -0.092 \\ -0.02 & -0.08 & 0.18 \end{pmatrix}$$

$$\text{So adjoint } (I\text{-}M) = \begin{pmatrix} 0.12 & -0.02 & -0.02 \\ -0.06 & 0.19 & -0.08 \\ 0 & -0.092 & 0.18 \end{pmatrix}$$

$$\text{And } (I-M)^{-1} = \frac{1}{0.054} \times \begin{bmatrix} 0.12 & -0.02 & -0.02 \\ -0.06 & 0.19 & -0.08 \\ 0 & -0.092 & 0.18 \end{bmatrix}$$

$$X = (I - M)^{-1} d = \frac{1}{0.054} \times \begin{bmatrix} 0.12 & -0.02 & -0.02 \\ -0.06 & 0.19 & -0.08 \\ 0 & -0.092 & 0.18 \end{bmatrix} \times \begin{bmatrix} 20 \\ 20 \\ 50 \end{bmatrix}$$

$$X = \begin{bmatrix} P \\ B \\ F \end{bmatrix} = \frac{1}{0.054} \times \begin{bmatrix} 0.12\times 20 - 0.02\times 20 - 0.02\times 50 \\ -0.06\times 20 + 0.19\times 20 - 0.08\times 50 \\ 0\times 20 - 0.09\times 20 + 0.18\times 50 \end{bmatrix} = \frac{1}{0.054} \times \begin{bmatrix} 1 \\ -1.4 \\ 7.2 \end{bmatrix} = \begin{bmatrix} 18 \\ -25.9 \\ 133 \end{bmatrix}$$

ii) The total output for packaging industry is less than the final demand. The total output of bakery is even negative showing that it requires more input to be able to produce final demand. The total output for flour is much more than the final demand since it supplies a lot of input to the bakery and packaging.

QUESTION 2

a)

i) These terms relate to kurtosis, which is the degree of flatness or peakedness of a frequency curve. Platykurtic means that a frequency curve is less peaked. The kurtosis is less than 3. Leptokurtic on the other hand means that the frequency curve is more peaked. The kurtosis is more than 3

Mesokurtic means that there is intermediate peakedness. This represents the normal curve. Kurtosis is equal to 3.

ii) Coeficient of skewness is a measure of skewness (lack of symmetry of a frequency distribution)

$$\text{Coefficient of skewness} = \frac{\text{Arithmetic mean} - \text{mod e}}{\text{Standard deviation}}$$

iii) Quatile deviation is a measure of dispersion expressed as follows

$$\text{Quartile deviation} = \frac{\text{Upper quartile} - \text{lower quartile}}{2}$$

b) A table to assist in determination of mean, median, mode and standard deviation is as follows.

Gross income	midpoint $x_i/1000$	Number f	fx_i	cumulative frequency	$f(x_i-x)^2$
0.5 - 5000.5	2.5005	2	5.0010	2	1037.6543
5000.5 - 10000.5	7.5005	15	112.5075	17	4740.7407
10000.5 - 15000.5	12.5005	25	312.5125	42	4081.7901
15000.5 - 20000.5	17.5005	28	490.0140	70	1693.8272
20000.5 - 25000.5	22.5005	33	742.5165	103	254.6296
25000.5 - 30000.5	27.5005	35	962.5175	138	172.8395
30000.5 - 35000.5	32.5005	38	1235.0190	176	1982.0988
35000.5 - 45000.5	40.0005	40	1600.0200	216	8669.7531
Total	**162.504**	**216**	**5460.1080**		**5838.6120**

i) The arithmetic mean AM $x = I \times \frac{\sum fxi}{\sum f} = 1000 \times \frac{5460.1080}{216} = 25{,}278.3$

Median determination

Q_2 frequency= $\frac{1}{2}(n+1) = \frac{1}{2}(216+1) = 108.1$ So Q_2 class interval is 25000.5-30000.5 with its frequency being 33. (seen from the above table where Q_2 = 108.1 is located).

Median= $l_{Q_2} + \frac{f_{Q_2} - f}{f_{Q_1}} \times C_{Q_2} = 25000.5 + \frac{108.5 - 103}{35} \times 5000 = 25{,}786$ Shillings

Where l_{Q_2} -lower boundary of Q_2 class

f_{Q_2} -Q_2 frequency

f_{Q_1} - Frequency of class after Q_2 class

f-Cumulative frequency at the start of Q_2 class

C_{Q_2} -class interval of the Q_2 class

Mode determination

The highest frequency is 40, so modal class is 35,000.5-45,000.5

Mode= $l_1 + \frac{f_0 - f_1}{(f_0 - f_1) + (f_0 - f_2)} \times C = 35000.5 + \frac{40-38}{(40-38)+(40-0)} \times 10000 = 35{,}476.70$ Shs.

Where l_1 -lower boundary of modal class

f_0 -frequency of modal class

f_1 - Frequency of class just before modal class

f_2- Frequency of class just after modal class

C -Interval of the modal class

ii) Coefficient of skewness= $\frac{\text{Arithmetic mean} - \text{mod e}}{\text{Standard deviation}} = \frac{25278.3 - 35476.7}{27030.6} = -0.3773$

Standard deviation = $I \times \sqrt{\frac{\sum f(x_i - \bar{x})^2}{\sum f}} = 1000 \times \sqrt{\frac{5835.6120}{216}} = 27030.60$

Since the coefficient is negative it shows the distribution is skewed to higher income earners. Meaning that air-conditioners are usually bought by high income earners.

QUESTION 3

a)

i) Compound event is the simultaneous occurrence of two or more events in connection with each other. It is an aggregate of simple events. The probability that the two events will occur is the joint probability of the events

ii) Mutually exclusive events are events that cannot occur simultaneously. Occurrence of one event precludes the occurrence of the other event. Two events A and B are mutually exclusive if they do not have any elementary outcome in common.

iii) Collectively exhaustive events exist when no other outcome is possible for a given experiment. That is, they include all possible outcomes. The sum of probabilities is equal to one.

iv) Equally likely events are events whereby each outcome is likely to occur the same way the other is likely to occur. That is, one event does not occur more than the other. Equal probability is assigned to each outcome.

v) Conditional probability is the probability of occurrence of an event given that another event has occurred.

b)

i) Number of technicians with salaries under Sh. 5,000 is 30 + 40=70. Total number of technicians is 500. So the probability of wages being below Sh 5,000 is $\frac{70}{500} = \frac{7}{50}$

ii) Number of technicians with salaries above Sh. 6,000 is 100+60+20=180 So probability of technicians having salaries above Sh 6,000 is $\frac{180}{500} = \frac{6}{25}$

iii) Number of technicians with salaries between Sh. 5,000 and Sh. 6,000 is 250

So probability of technicians having salaries between Sh. 5,000 and Sh. 6,000 is $\frac{250}{500} = \frac{1}{2}$

QUESTION 4

i) Goodness of fit test is a test on how well empirical distribution(obtained from sample data) can fit theoretical distribution (like normal, Poisson or binomial distributions) using the χ^2 test. Accountants can use it to determine whether a given age-debtors distribution can be approximated by a given function. Also while forecasting past data or surveyed data can be compared with assumed distribution to come up with a conclusion that the distribution function represents the forecast Accountants can also come up with appropriate wage/salary given that a certain distribution exists between staff turnover and salary/wages

ii) A table to aid in calculation of distribution and x^2 is as follows:

No. of people changing	Observed values			Poison distribution	$\frac{(f_0 - f_e)^2}{f_e}$
x	O	f_0	f_0x	f_e	
0	8	0.073	0.000	0.039	0.030
1	18	0.164	0.164	0.126	0.012
2	19	0.173	0.345	0.204	0.005
3	20	0.182	0.545	0.222	0.007
4	16	0.145	0.582	0.180	0.007
5	12	0.109	0.545	0.117	0.001
6	8	0.073	0.436	0.064	0.001
7	4	0.036	0.255	0.030	0.002
8	3	0.027	0.218	0.012	0.019
9	2	0.018	0.164	0.004	0.044
Total	**110**	**1**	**3.255**		**0.127**

Poisson distribution $f_e = \frac{e^{-\lambda}\lambda^x}{x!}$ and $f_o = \frac{\text{observed value}}{\text{total value}}$

Mean $\lambda = \frac{\sum f_0 x}{\sum f_0} = 3.255$

$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e} = 0.127 < \chi^2_{0.05,8df} = 15.5$ so the Poisson distribution fits well for the data.

QUESTION 5

a)

i) Scatter diagram is a plot of a distribution in its ungrouped form on a graph
ii) Bivariate distribution is a distribution of two variables
iii) Positive correlation occurs when movement of one variable in one direction causes the other variable to move in the same direction
iv) Confidence interval is the limit at which a parameter or the linear regression itself is taken to represent a given distribution
v) Autocorrelation occurs when a series' errors or disturbance covariance is not equal to zero so the least squares estimated are not the best linear unbiased estimates.

b)

i) A table to aid in calculation of the regression line equation is a s follows

Year	Business turnover Sh millions x	Staff requirement y	xy	x^2
1993	45	2,600	117,000	2,025
1994	50	3,000	150,000	2,500
1995	60	3,100	186,000	3,600
1996	75	3,530	264,750	5,625
1997	80	3,850	308,000	6,400
1998	110	4,300	473,000	12,100
1999	150	5,870	880,500	22,500
2000	170	7,150	1,215,500	28,900
Totals	740	33,400	3,594,750	83,650

$y = a + bx$ Where x-Sh million
n- Number of years considered

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{8\times 3594750 - 740\times 33400}{8\times 83650 - (740)^2} = 33.24$$

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n} = \frac{33400}{8} - 33.24\times\frac{740}{8} = 1100.3$$

So $y = 1100.3 + 33.24 \times x$

ii) Staff requirement when business turnover is x = Sh 200 million is as follows $y = 1100.3 + 33.24 \times 200 = 7748.3 \approx 7748$

QUESTION 6

a) The underlying assumptions are:
Proportionality/linearity-variables forming the basis of the problem vary in direct proportion with the level of activity.
Certainty/deterministic-coefficients in the objective and constraints are known with certainty Divisibility- fractional levels for variables in objective and constraints are allowed.
Additive- the total contribution of all activities are identical to the sum of the contribution of each activity taken individually
Time factor is ignored

b)

i) Let x_1, x_2 and x_3 be the quantities of ordinary chair, baby cot and executive chairs. Z-profit from sale of furniture

Objective function

$$z = 26x_1 + 35x_2 + 50x_3$$

Constraints:

$10x_1 + 12x_2 + 20x_3 \leq 300$ cubic meters of wood

$3x_1 + 5x_2 + 6x_3 \leq 118$ Hours lathe time

$2x_1 + 3x_2 + 5x_3 \leq 90$ Hour's technicians labour

$x_1, x_2, x_3 \geq 0$ Non-negativity of variables

ii) From the computer solution the units required to maximise profit are: x_1= 6 ordinary chairs

x_2= 20 baby cots

x_3= 0 executive chairs

The maximum profit is Shs. 856

iii) Yes, the time required for technicians is not completely used up. There are 18 hours remaining as slack. For used up resources, one can pay:

Wood ≈ Sh. 1.80 for any additional cubic

meter Lathe ≈ Sh. 2.70 for any additional hour

QUESTION 7

a)

i) N-person game is a game involving n persons

ii) Saddle point is that position in the payoff matrix where maximum of row minima is equal to the minimum of column maxima

iii) A strategy is the number of competitive actions, choices or alternatives that are available for a player.

iv) Rules of dominance are the guidelines that assist to identify an inferior strategy from others. The rule is that if all elements in a column (or row) are greater than/equal to corresponding element in another column (or row) then that column (or row) is eliminated.

b)

i) If Peter expands and Ngomongo expands too, Peter will have additional profit per day of Sh1000, otherwise if Ngomongo does not expand then there will be no additional profit to Peter. In case Peter does not expand and Ngomongo does, then he will loose Sh500 per day, otherwise if Ngomongo does not expand too then he will be getting Sh500 profit.

ii) The payoff table is as follows:

			Ngomongo Expand	Do not expand		
			y	1-y	Row minima	
Peter	Expand	x	1000	0	0	⇐Maximum
	Do not expand	1-x	-500	500	-500	
	Column minima		1000	500		
			⇑ Minimum			

There is no saddle point since the minimum of colum maxima is not the maximum of row minimum.

So mixed strategies will be used to find the optimum solution.

The value of the game can be determined as follows

x-probability of Peter expanding

y-probability of Ngomongo expanding

Expected value of the game $E(x,y) = 1000xy + 0 \times x(1-y) - 500(1-x)y + 500(1-x)(1-y)$

$= 1000xy - 500(1-x)y + 500(1-x)(1-y)$

differentiating E(x,y) with respect to x and equating to zero gives

$$\frac{\partial(E(x,y))}{\partial x} = 1000y + 500y - 500(1-y) = 0 \Rightarrow 2000y = 500 \Rightarrow y = 0.25$$

differentiating E(x,y) with respect to y and equating to zero gives

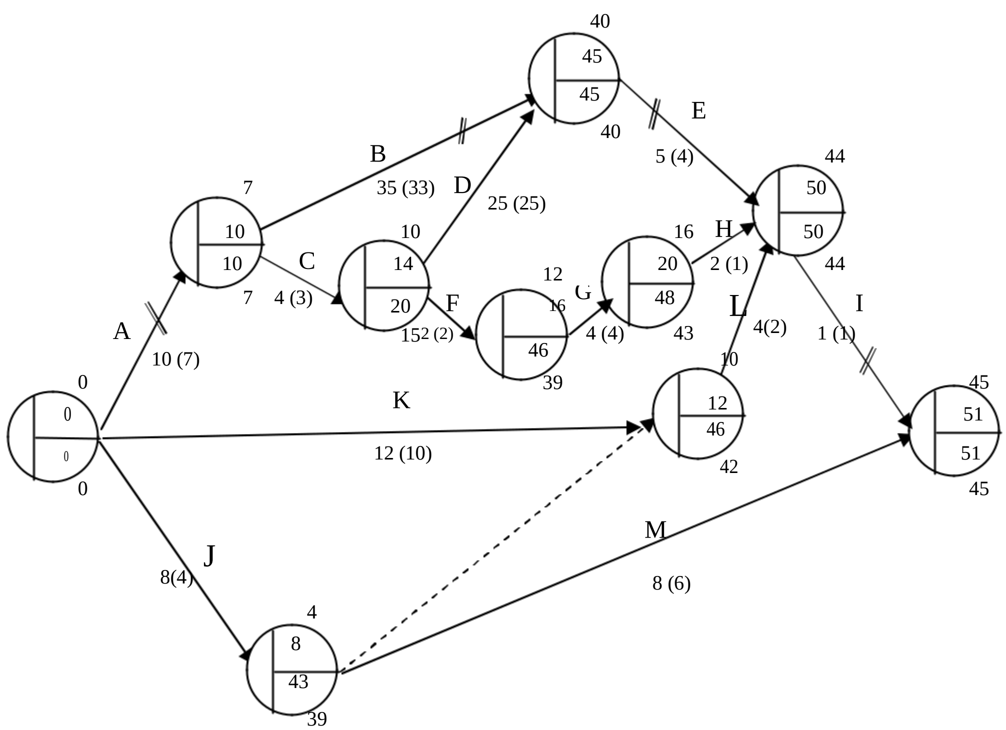
$$\frac{\partial(E(x,y))}{\partial y} = 1000x - 500(1-x) - 500(1-x) = 0 \Rightarrow 2000x = 1000 \Rightarrow x = 0.5$$

The probability of Peter expanding x=0.5 and for not expanding 1-x=0.5 The probability of Ngomongo expanding y=0.25 and not expanding 1-y=0.75

The value of the game when Peter and Ngomongo play the strategies as shown is obtained as follows. Solving the values for x and y in the expected value equation E(x,y) =1000×0.5×0.25-500×(1-0.5)×0.25+500×(1-0.5)(1-0.25) = Sh 375

QUESTION 8

i) The network diagram is as follows



The activity durations in brackets are the crash times. The event times on top and below the events are the event times obtained from crash times.

Critical path is A-B-E-I

ii) The minimum time is 45 weeks.

The minimum cost is determined by crashing all the activities that there is a cost reduction in the crashing. These are identified as activity E,K,L, and M.

Total cost for the project without crashing is Sh 185,000. Crashing E,K,L, and M means that the cost will reduce from 185000 to (185000-(1×500+2×1000+2×2000+1000×2)=176500

iii) For the activities the ratio $r = \dfrac{\text{Crashcost} - \text{Normalcost}}{\text{Normal time} - \text{Crash time}}$ is given as

Activity	Ratio r
A	666.7
B	1000
C	1000
D	0
E	500
F	0
G	0
H	10000
I	0
J	3000
K	-1000
L	-2000
M	-1000

The cost schedule for the activities is as follows

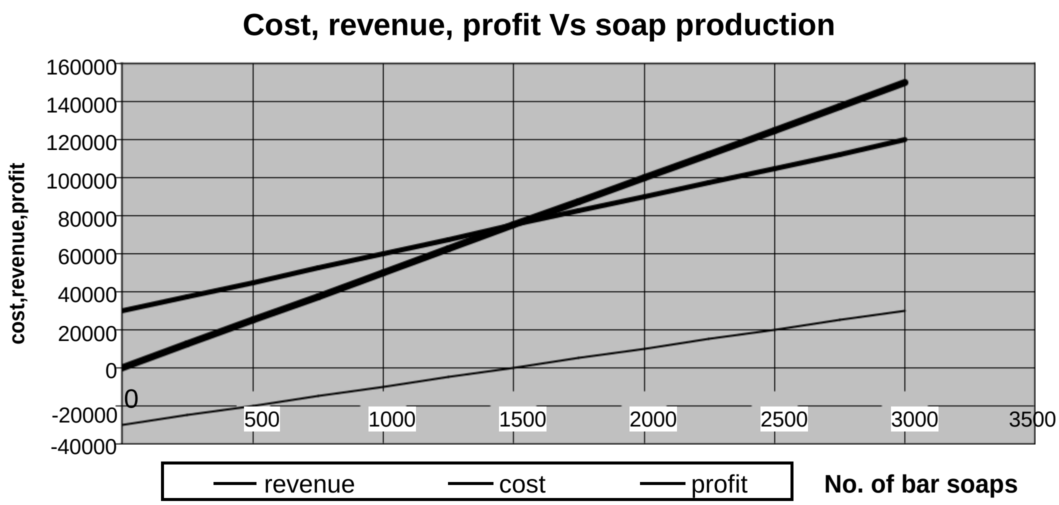
Comment	Duration	Direct cost	Opportunity cost	Total cost
Normal time	51	185000	11000	196000
Crash K,L,M	51	177000	11000	188000
Crash E	50	176500	10000	186500
Compress A	49	177166.7	9000	186166.7
Compress A	48	177833.4	8000	185833.4
Crash A	47	178500.1	7000	185500.1
Compress B	46	179500.1	6000	185500.1
Crash B	45	180500.1	5000	185500.1

Yes it is advisable to crash the project to 45 weeks since the project will be running at minimum cost of Sh.185500.1 in addition to the reduced time.

Paper 2

QUESTION 1

a) Cost function=30x+3000 Revenue function=50x Profit=revenue-cost=50x-30x-30000=20x-30000 Where x - number of bar soaps.



i) Break-even point is where profit is equal to zero or where cost function line crosses the revenue function line. From the graph the break-even point is x=1500 bar soaps.

ii) From the graph, selling 2500 bar soaps the profit will be KSh 20,000

b)

Revenue is maximized when the first partial derivatives of revenue are equated to zero

$$\frac{\partial R}{\partial x} = 8 + 2y - 2x = 0 \Rightarrow 2y - 2x = -8 \quad (1) \qquad \frac{\partial^2 R}{\partial x^2} = -2 \Rightarrow \text{Maximum point}$$

$$\frac{\partial R}{\partial y} = 5 + 2x - 4y = 0 \Rightarrow -4y + 2x = -5 \quad (2) \qquad \frac{\partial^2 R}{\partial y^2} = -4 \Rightarrow \text{Maximum point}$$

Adding equation (1) and (2)

-2y=-13⇒y=6.5 ‘000’ and x=2.5 ‘000’

$$\frac{\partial^2 R}{\partial y \partial x} = 2 < \frac{\partial^2 R}{\partial x^2} \times \frac{\partial^2 R}{\partial y^2} = -2 \times -4 = 8 \Rightarrow \text{showing that the point is maximum}$$

c) Sampling is a process of examining a representative number of items out of a whole population.

- Systematic sampling involves selecting every nth item after selecting the first item randomly.
- Stratified sampling involves taking random samples from within a group in the population that each group bears to the population as a whole.
- Multi-stage sampling is similar to stratified sampling except that the groups are geographically based.
- Cluster sampling involves selecting a few areas at random and every single item in the area is interviewed.
- Quota sampling involves choosing on the spot up to a given quota. An interviewer selects interviewee from given categories up to a given quota.

QUESTION 2

a)

i) Markov process is a sequence of events in which the probability of occurrence for one event depends upon the preceding event. It is time-based process. For example a patients state today depends on previous days state

ii) Cyclic chain is one that repeats itself in a deterministic manner. The transition matrix has one's in two or more rows that form a closed path among cycle states. Example is a machine operation that repeats itself

iii) Absorbing state is one that cannot be left once entered. It has a transition probability of one to itself and zero to other states. Example includes the payment of a bill, sale of a capital asset or termination of an employee

iv) State transition matrix is a rectangular array that summarises the transition probabilities for a given Markov process. Transition probabilities are the probabilities of occurrence of each event depending on the state of the generator

v) Steady state is the condition that in the long run period of time a system settles or stabilizes to.

b)

i) February 2002 market share = January 2002 market share×Transition matrix

$$= (0.4 \quad 0.3 \quad 0.3) \times \begin{matrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{matrix}$$

$$= (0.4\times 0.5 + 0.3\times 0.2 + 0.3\times 0.1 \quad 0.4\times 0.4 + 0.3\times 0.7 + 0.3\times 0.2 \quad 0.4\times 0.1 + 0.3\times 0.1 + 0.3\times 0.7)$$

$$= (0.2 + 0.06 + 0.03 \quad 0.16 + 0.21 + 0.06 \quad 0.04 + 0.03 + 0.21)$$

$$\underset{(CU\ CG\ AF)}{=} (0.29 \quad 0.43 \quad 0.28)$$

For March 2002 market share = February market share× Transition matrix

$$= (0.29 \quad 0.43 \quad 0.28) \times \begin{matrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{matrix}$$

$$=(0.29\times0.5+0.43\times0.2+0.28\times0.1 \quad 0.29\times0.4+0.43\times0.7+0.28\times0.2 \quad 0.29\times0.1+0.43\times0.1+0.28\times0.7)$$

$$= (0.145 + 0.086 + 0.028 \quad 0.116 + 0.30 + 0.056 \quad 0.029 + 0.043 + 0.196)$$

$$\underset{(CU\ CG\ AF)}{=} (0.259 \quad 0.417 \quad 0.268)$$

ii) Steady state Situation

At steady state

XT=X Taking CU - x, CG - y and AF - Z

$$(CU \quad CG \quad AF) \times \begin{matrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{matrix} = (CU \quad CG \quad AF)$$

And $x+y+z=1 \Rightarrow x=1-y-z$ (1)

From the matrix multiplication

$0.5x+0.2y+0.1z=x$ (2)

$0.4x+0.7y+0.22z=x$ (3)

Substituting equation (1) into equations (2) and (3)

$0.5-0.5y-0.5z+0.2y+0.1z=1-y-z$

$0.7y+0.6z=0.5$ (4)

$0.4-0.4y-0.4z+0.7y+0.2z=y$

$0.7y+0.2z=0.4$ (5)

Solving equations (4) and (5) simultaneously

$0.4z=0.1 \Rightarrow z=0.25$

$$y = \frac{0.5 - 0.15}{0.7} = \frac{0.35}{0.7} = 0.5$$

$x=1-0.5.025=0.25$

Steady state $(0.25 \;\; 0.5 \;\; 0.25)$

QUESTION 3

a)

i) Price relative is an index number that compares prices of a given commodity for a given period in relation to its price in another period.

ii) Fixed base method compares prices of a commodity for a given period in relation to price of the commodity in a base period.

iii) In chain base method, the base is the price of the previous period. It is not fixed to a particular period

iv) Weighted index number is one that appropriate weights have been added to reflect relative importance of commodities. This makes index numbers be reliable.

v) Quantity index number is one that shows the change from one period to the other of quantities involved.

b) The arithmetic mean of price is $= \frac{\sum P_i}{n} = \frac{100 + 130 + 180}{3} = \frac{410}{3} = 136.7$

Geometric mean $= \sqrt[n]{P_1 \times P_2 \times P_3} = \sqrt[3]{100 \times 130 \times 180} = 132.8$

c) Let P_1- selling price in period 1

P_0- selling price in base period

Then $P_0 = 0.5P_0 + 0.4P_0 + 0.1P_0$

And $P_1 = (0.5P_0 \times 1.2) + (0.4P_0 \times 1.1) + 0.1P_1 = 0.6P_0 + 0.44P_0 + 0.1P_1$

$\Rightarrow 0.9P_1 = 1.04P_0 \Rightarrow \frac{P_1}{P_0} = \frac{1.04}{0.9} = 1.156$

So price change $= \frac{P_1}{P_0} \times 100 = 115.6$ The percentage increase in selling price is 15.6%

QUESTION 4

a) A Venn diagram is a simple diagrammatic representation of a well defined list, collection or class of objects. (Set)

It can be used to determine probability of a given event when a sub set is taken out of a set. The ratio of the subset to the set gives probability of the given subset

b) i) Let A- the set employees with salary increment =300

B-the set employees promoted =100

U-universal set =1,500

Given that $A \cap B = 50$ A Intersection B

Then $B \cap \overline{A} = B - (A \cap B) = 100 - 50 = 50$

Read as B intersection not A = B minus A

Intersection B The employees who were only promoted.

So probability $= \frac{50}{1500} = \frac{1}{30}$

ii) And $U - (BUA) = [U - B + A - (AUB)] = 1500 - (300 + 100 - 50) = 1150$

The employees who did not get promoted or increment.

So probability= $\frac{1150}{1500} = \frac{23}{30}$

c)

i) A random variable is a numerically valued function defining a set of all possible outcomes of an experiment that depends on chance

ii) a) This is a case of binomial distribution

Probability of defective q=0.05

Probability of no defective p=1-0.05=0.95

So probability of no defective given that n=50 is as follows

$P_0 = {}^nC_i p^n q^i$ where i=0 $\Rightarrow P_0 = \frac{50!}{0! \times 50!} \times 0.95^{50} \times 0.05^0 = 0.95^{50} = 0.0769$

b) Probability of not more than 4 being defective= Probability of at least having 4 defectives $P_4 = (P_0+P_1+P_2+P_3+P_4)$

$= ({}^{50}C_0 p^{50} q^0 + {}^{50}C_1 p^{49} q^1 + {}^{50}C_2 p^{48} q^2 + {}^{50}C_3 p^{47} q^3 + {}^{50}C_4 p^{46} q^4)$

$= \frac{50!}{0! \times 50!} \times 0.95^{50} \times 0.05^0 + \frac{50!}{1! \times 49!} \times 0.95^{49} \times 0.05^1 + \frac{50!}{2! \times 48!} \times 0.95^{48} \times 0.05^2 + \frac{50!}{3! \times 47!} \times 0.95^{47} \times 0.05^3 + \frac{50!}{4! \times 46!} \times 0.95^{46} \times 0.05^4$

$= 0.95^{50} + 50 \times 0.95^{49} \times 0.05^1 + 1225 \times 0.95^{48} \times 0.05^2 + 19600 \times 0.95^{47} \times 0.05^3 + 230300 \times 0.95^{46} \times 0.05^4$

$= (0.0769 + 0.202 + 0.2611 + 0.2199 + 0.1360)$

=0.896 = probability of meeting guaranteed quality

QUESTION 5

a) Quantitative techniques are the mathematical techniques used to analyse past data to come up with past patterns, so as to guide about the future. These include moving average, exponential smoothing and time series decomposition.

- Moving averages method removes cyclical movements by getting averages of past data. The average can be weighted or un-weighted. The averages are changed with additional data.
- Exponential smoothing on the other hand uses weights that decrease exponentially with time. The new forecast uses the previous forecast and a proportion of the forecast error (difference between current observation and previous forecast. The proportion is the smoothing constant.
- Time series decomposition involves deseasonalising data by use of moving averages then fixing a relationship that can be used to predict the future

  Qualitative techniques use other techniques like judgment, intuition, experience and flare. It is used where past data is not available and where long term forecast is required. These methods include Delphi method, market research and historical analogy.
- Delphi method is a technique designed to obtain expert consensus for a particular forecast. Experts independently answer a sequence of questionnaires in which responses of one questionnaire is used to produce the next questionnaire. So information is passed to other experts whereby judgments are refined with more information and experience.
- Market research involves the surveys of opinions, market data analysis and questionnaires designed to get reaction of market to a particular product or design. This is mainly important in short term.
- Historical analogy involves looking at data of a similar product being studied. The similar product is analysed through its lifecycle.

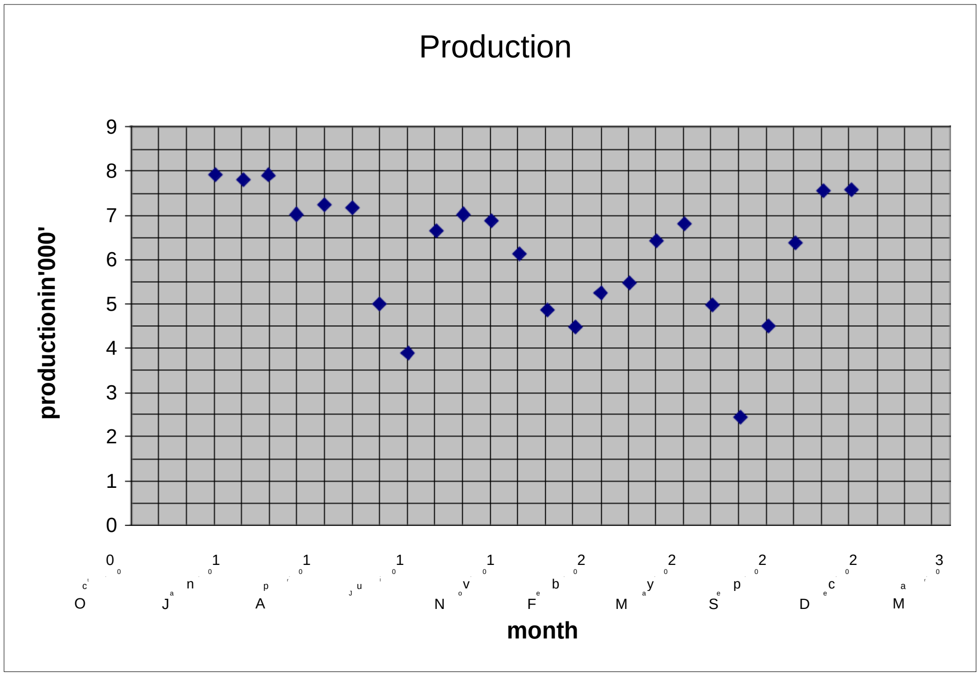
b)

i) Time series decomposition is the breaking down of given data to come up with a given relationship that can predict the future. The various characteristics (trend, seasonal variation, cyclical variation and residual variation) are separated. The two ways of forecasting are by use of multiplicative and additive models.

In multiplicative model the steps involved include the following

- Deseasonalise the data by separating of trend and seasonal variation (using moving averages)
- Calculate the trend using least squares method
- Estimate the dependent variables from the regression formula
- Calculate the percentage variation of the dependent variable in each period
- Average this percentage variation of dependent variable to give average seasonal variation
- Forecast is made by multiplying trend with the percentage variation

ii) Plotting these values on a scatter diagram gives the following



Because the variations from the trend are increasing with time, the multiplicative model is appropriate. First step is to deseasonalise the data using a three months moving average

Month	Production A	3pt moving average T	Ratio A/T=S×E	Seasonal S	De-season A/S=T×E	Trend T=7.18-.085×x	x	x^2	xy
Jan-01	7.92			0.9434	8.40	7.095	1	1	8.3950
Feb-01	7.81	7.8800	0.9911	0.9784	7.98	7.01	2	4	15.9643
Mar-01	7.91	7.6200	1.0381	1.0519	7.52	6.925	3	9	22.5593
Apr-01	7.03	7.4000	0.9500	0.9937	7.07	6.84	4	16	28.2989
May-01	7.25	7.4467	0.9736	1.0519	6.89	6.755	5	25	34.4602
Jun-01	7.17	6.7000	1.0701	1.1770	6.09	6.67	6	36	36.5519
Jul-01	5.01	5.6100	0.8930	1.0422	4.81	6.585	7	49	33.6503
Aug-01	3.9	6.1533	0.6338	0.6298	6.19	6.5	8	64	49.5431
Sep-01	6.64	7.1967	0.9226	0.9169	7.24	6.415	9	81	65.1771
Oct-01	7.03	7.2767	0.9661	1.0286	6.83	6.33	10	100	68.3426
Nov-01	6.88	6.9800	0.9857	1.0989	6.26	6.245	11	121	68.8689
Dec-01	6.14	6.3067	0.9736	1.0873	5.65	6.16	12	144	67.7633
Jan-02	4.86	5.7533	0.8447	0.9434	5.15	6.075	13	169	66.9694
Feb-02	4.48	5.8867	0.7610	0.9784	4.58	5.99	14	196	64.1027
Mar-02	5.26	6.2200	0.8457	1.0519	5.00	5.905	15	225	75.0075
Apr-02	5.48	6.6067	0.8295	0.9937	5.51	5.82	16	256	88.2378
May-02	6.42	7.0533	0.9102	1.0519	6.10	5.735	17	289	103.7512
Jun-02	6.82	6.5733	1.0375	1.1770	5.79	5.65	18	324	104.303
Jul-02	4.98	5.1167	0.9733	1.0422	4.78	5.565	19	361	90.7896
Aug-02	2.45	4.9600	0.4940	0.6298	3.89	5.48	20	400	77.8081
Sep-02	4.51	6.2700	0.7193	0.9169	4.92	5.395	21	441	103.2952
Oct-02	6.38	7.2833	0.8760	1.0286	6.20	5.31	22	484	136.4519
Nov-02	7.55	7.6867	0.9822	1.0989	6.87	5.225	23	529	158.0217
Dec-02	7.59			1.0873	6.98	5.14	24	576	167.5321
				Total	146.72		300	4900	1735.845

The seasonal component S is determined as follows

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
2001		0.9911	1.0381	0.9500	0.9736	1.0701	0.8930	0.6338	0.9226	0.9661	0.9857	0.9736	
2002	0.8447	0.7610	0.8457	0.8295	0.9102	1.0375	0.9733	0.4940	0.7193	0.8760	0.9822		
Ave S	0.8447	0.8761	0.9419	0.8897	0.9419	1.0538	0.9332	0.5639	0.8210	0.9210	0.9839	0.9736	10.7447
AdjS	0.9434	0.9784	1.0519	0.9937	1.0519	1.1770	1.0422	0.6298	0.9169	1.0286	1.0989	1.0873	12.0000

From the deseasonalised data, using least square method, the trend $Production_T = a + b \times month$

Let x = month

y = deseasonalised data

Where $b = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \dfrac{24\times 1735.845 - 300\times 146.72}{24\times 4900 - (300)^2} = -0.085$

$a = \dfrac{\sum y}{n} - b\dfrac{\sum x}{n} = \dfrac{146.72}{24} - (-0.085)\times\dfrac{300}{24} = 7.180$

So that $Production_T = 7.18 - 0.085\times month$ in '000'

The final graph raw data, deseasonalised data and trend will look as follows:

NOTES: To get 3 point moving average

For example for Feb `01 = $\dfrac{\text{Jan 01 + Feb 01 + Mar 01}}{3}$

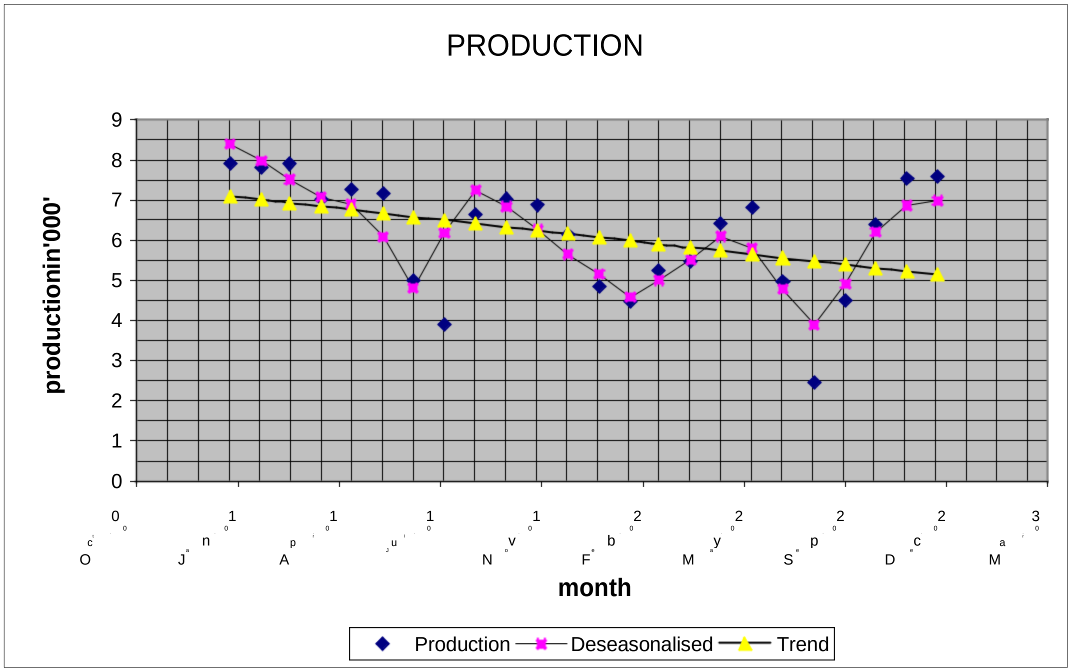
$$= \frac{7.92 + 7.81 + 7.91}{3} = 7.88$$

$$\frac{\text{Feb 01 + Mar 01 + Apr 01}}{3}$$

Mar `01 = $= \dfrac{7.81 + 7.91 + 7.03}{3} = 7.62$

A/T = S×E = seasonal factor × error term

To deseasonalise the data, the seasonal component has to be determined. From the given data, a multiplicative model is appropriate.
A three-month moving average is chosen since centering is not required.
The ratio A/T=S×E (A-actual data, T-trend, S-seasonal factor and E-error term) is then determined by dividing the actual data by the moving average data. The A/T ratio is then placed for each month and year as per the second table to average out the seasonal factor. Since the total of the averages does not equal to 12 (the number of months or seasons), adjustment are done by a factor of 12/ total average=12/10.7447
The seasonal factor is then used to deseasonalise the actual data by dividing the actual data A by seasonal factor S.
The trend from calculations is then determined by using the deseasonalised data (which is taken to be y while x is taken to be the time in months) x^2 and x×y can then be determined to assist in coming up with the trend equation.
Forecast is made of the trend × seasonal factor.
So for May 2003, the trend is determined and multiplied by seasonal factor of May (1.051) to obtain the forecast.
Notice: The trend values given are not required. Also the graph is not required. They have just been reproduced to show the movement from raw data to the trend values. Critical path is B-D-E-G-H-J-K



The production forecast of May 2003 (month=29) is:

$$\text{Pr oduction}_T = 7.18 - 0.085 \times 29 = 4.715 \times 1000 = 4,715 \text{ Units}$$

QUESTION 6

a) A dual problem is a linear programming problem of maximisation (or minimisation) that is unique to a minimisation (or maximisation) problem (which is the primal). The dual uses the same data and numerical values of objective function and gives the same solution as the primal

b)

i) Let x_1, x_2, x_3 be the 5 litre quantity of lemonade, grapefruit and orangeade and z be the revenue to be maximised.

Objective function

$$z = 37.5x_1 + 40x_2 + 42.5x_3 \text{ maximise turnover}$$

Constraints

$2x_1 \leq 2500$ Dozen of lemons

1

$1\frac{1}{2}x_3 \leq 750$ Dozen of oranges

$2x_1 + 1\frac{1}{2}x_2 + 1\frac{1}{2}x_3 \leq 5000$ Kg of sugar

$2x_1 + 1\frac{1}{2}x_2 + x_3 \leq 3000$ ounces of citric acid

$x_1, x_2, x_3 \geq 0$ Non-negativity of variables

ii) Dual of the problem is as follows

Let y_1, y_2, y_3, y_4, y_5 be the quantities of lemon, grapefruit, oranges, sugar and citric acid respectively. Objective function

$$z = 2500y_1 + 2000y_2 + 750y_3 + 5000y_4 + 3000y_5 \text{ minimise usage}$$

Constraints		
	$2y_1 + 2y_4 + 2y_5 \geq 37.5$	price of 5 litre lemonade
	$2y_2 + 1\frac{1}{2}y_4 + 1\frac{1}{2}y_5 \geq 40$	price of 5 litre grapefruit
	$1\frac{1}{2}y_3 + 1\frac{1}{2}y_4 + y_5 \geq 42.5$	price of 5 litre orangeade

$y_1, y_2, y_3, y_4, y_5 \geq 0$ Non-negativity of variables

iii) The quantities to be manufactured in May 2002 that maximise turnover are:
250 (5 litres) lemonade
1333.33 (5 litres) grapefruit
500 (5 litres) orangeade
All are read from the dual prices of the dual problem. The maximum turnover is Sh. 83958.30

iv) The resources not used up are read from the reduced gradient. It shows that there is slack in lemons of 2000 dozen and sugar of 1750 kg.

QUESTION 7

a) Laplace criterion is a decision rule that expected payoff is calculated where each alternative is assigned equal probabilities. The best alternative is that with the highest expected payoff. Hurwicz criterion on the other hand is a decision rule where there is a weighted average of the best and worst payoffs of each alternative. Laplace criterion is different from Hurwicz criterion because of the equal probabilities and the other uses an average depending on the decision maker's view of risk

b) Payoffs table in Million shillings for the various price strategies is follows i)

		Variable cost		
		High	Most likely	Low
		1900	1700	1400
Client fee	3600	26.775	29.925	34.65
	4000	27.5625	30.1875	34.125
	4400	26.25	28.35	31.5

The amounts of payoffs in the table are determined from the expression:
Contribution per client per day×50×350×occupancy level
e.g. for a charge of Sh.3,600 and variable cost of 1,700, the payoff =

(3600-7700) × 50 × 350 × 90%
=Sh.29,925,000

ii) Maximax decision rule- maximize the maximum payoffs

		Maximum payoff	
Client fee	3600	34.65	⇐Best strategy
	4000	34.125	
	4400	31.5	

Maximin decision rule- maximize the minimum payoff

		Minimum payoff	
Client fee	3600	26.775	
	4000	27.5625	⇐Best strategy
	4400	26.25	

Regret table is as follows in million shillings. The strategy that minimises the maximum regrets is the best.

		Variable cost				
		High	Most likely	Low	Maximum	
		1900	1700	1400		
Client fee	3600	0.7875	0.2625	0	0.7875	
	4000	0	0	0.525	0.525	⇐Best strategy
	4400	1.3125	1.8375	3.15	3.15	

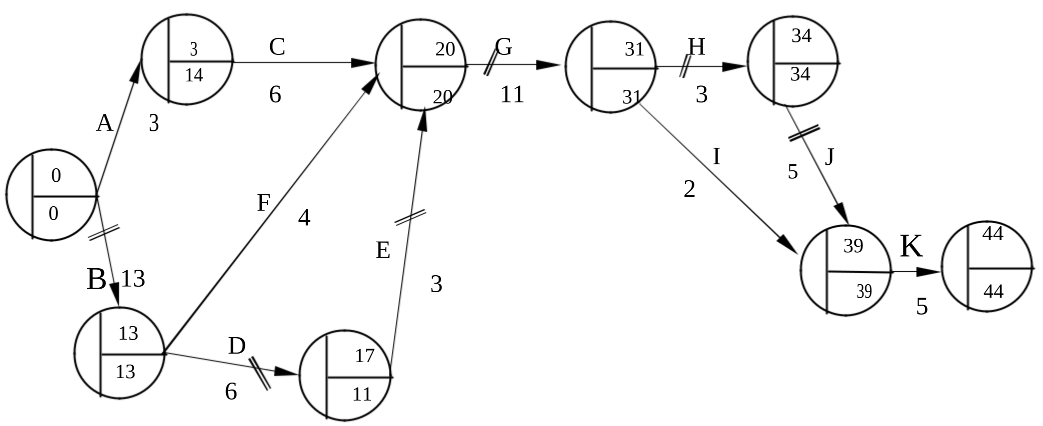
For example regret for a charge of Sh.4,000 and variable cost of Sh.1,400 =
34.65 - 34.125 = 0.525 million shillings

QUESTION 8

a)

Activity	L	O	P	M	SD	SD_2
A	6	4	8	3.0	0.3333	0.1111
B	24	20	40	13.0	1.6666	2.77756
C	10	8	24	6.0	1.3333	1.77764
D	8	4	12	4.0	0.6666	0.44441
E	6	6	6	3.0	0	0
F	8	6	10	4.0	0.3333	0.1111
G	20	16	36	11.0	1.6666	2.77756
H	6	4	8	3.0	0.3333	0.1111
I	4	4	4	2.0	0	0
J	10	8	12	5.0	0.3333	0.1111
K	8	4	24	5.0	1.6666	2.77756
					Total	10.9991

b)



Critical path B-D-E-G-H-J-K

Since the project is expected to take 44 days then the actual cost is 44 days × Sh10,000=Sh440,000 which is within the budget

c) The probability of the project being completed 4 days before expected completion time (that is 40 days) is determined as follows

$$P(t \leq 40) = P\ z = \frac{40\text{-}44}{\sigma_P} = P\ z = \frac{-4}{3.3165} = P(z = -1.2061)$$

From normal distribution table at z=1.21 the probability is 0.5-0.3869=0.1131 11.31%

Paper 3

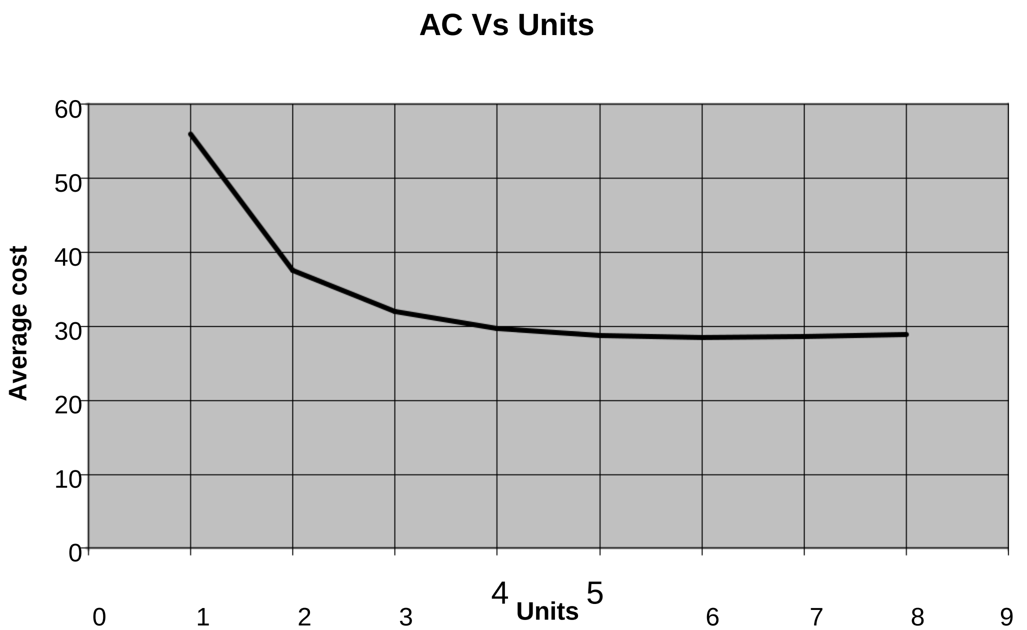
QUESTION 1

a) When faced with single variable the differentiation of a function of cost, revenue or profit will give optimum conditions when equated to zero. Depending on what is to be optimized a manager can be able to obtain the quantity of variable that will optimize a given function.
For example, a fund manager can check behaviour of a given stock and determine the optimum time to purchase the stock, by differential calculus.

b)

i) Average cost $AC = \frac{\text{Cost}}{\text{Units}} = x + 16 + \frac{39}{x}$

The graph of cost vs. units is as follows



ii) Total revenue TR $= Px$

$= (x^2 - 24x + 117)x$

$= x^3 - 24x^2 + 117x$

iii) $\frac{d(TR)}{dx} = 3x^2 - 48x + 117 = 0$ Solving for x

$$x = \frac{48 \pm \sqrt{48 - 4 \times 3 \times 117}}{2 \times 3} = \frac{48 \pm 30}{6} = 13 \text{ or } 3$$

When x=13

$TR = 13^3 - 24 \times 13^2 + 117 \times 13 = -338$

When x=3

$TR = 3^3 - 24 \times 3^2 + 117 \times 3 = 162$

So the quantity that maximizes revenue is x=3. The price to be charged then is $P = 3^2 - 24 \times 3 + 117 = 54$ million shillings

iv) At x=3

$$\frac{dp}{dx} = 2x - 24 = 6 - 24 = -18$$

$\frac{P}{x} = x - 24 + \frac{117}{x} = 13 - 24 + \frac{117}{3} = 18$

So elasticity of demand = $\left(\frac{P}{x}\right) \times \frac{1}{(dP/dx)} = 18 \times \frac{1}{-18} = -1$

v) Profit =TR-TC

=x^3-24x^2+17x-(x^2+16x+39)

=x^3-25x^2+101x-39

To maximize profit the derivative of profit with respect to x is equal to zero

$\frac{d(\text{Profit})}{dx} = 3x^2 - 50x + 101 = 0$ Solving for x

x = [redacted] = 14.31 or 2.35

2 × 3 6

When x=14.31

Profit=14.31^3-25×14.31^2+101×14.31-39=-782.7 million shillings

When x=2.35

Profit=2.35^3-25×2.35^2+101×2.35-39=73.27 million shillings

So the price that maximizes profit is found by substituting x=2.35 into the price equation P=2.35^2-24×2.35+117=66.1225 million shillings.

QUESTION 2

a) First probabilities associated with retentions, gains and losses are determined. From this a state transition matrix is drawn. Secondly using the state transition matrix obtained and the current state, the future state in one period (t+1)is obtained as follows

State$_{(t+1)}$=State $_t$×(State transition matrix)

Thirdly using the state(t+1) and state transition matrix the state(t+2)is obtained as follows State$_{(t+2)}$=State$_{(t+1)}$×(State transition matrix)

Lastly to obtain the steady state the following is solved for the unknowns in the matrix algebra as follows

X×(transition matrix)=X (1)

Sum of the vector elements=1 (2)

Where x is the starting state.

b)

i) The elements mean the following

- 0.84- of those debtors who were rated as excellent, 84% remained exclent the following month
- 0.2- of those debtors who were rated as good, 20% were rated s poor the following month
- 0.18-of those debtors who were rated as poor, 18% were rated as good the following month
- 0.16-of those who had been rated as excellent 16% were rated as good the following month

ii) September make-up=August make-up×transition matrix

$$= (30\ \ 50\ \ 20) \times \begin{pmatrix} 0.8 & 0.18 & 0.02 \\ 0.2 & 0.75 & 0.05 \\ 0 & 0.16 & 0.84 \end{pmatrix} \times 1000$$

$$=(30\times0.8+50\times0.2+20\times0 \quad 30\times0.18+50\times0.75+20\times0.16 \quad 30\times0.02+50\times0.05+20\times0.84)\times1000$$

$$= (34\ \ 46.1\ \ 19.9)\times1000$$

October make-up=September make-up×transition matrix

$$= (34 \ \ 46.1 \ \ 19.9)\times \begin{pmatrix} 0.8 & 0.18 & 0.02 \\ 0.2 & 0.75 & 0.05 \\ 0 & 0.16 & 0.84 \end{pmatrix} \times 1000$$

$$= (34\times0.8+46.1\times0.2+19.9\times0 \ \ 34\times0.18+46.1\times0.75+19.9\times0.16 \ \ 34\times0.02+46.1\times0.05+19.9\times0.84)\times100$$

$$= (36.42 \ \ 43.879 \ \ 25.821)\times1000$$

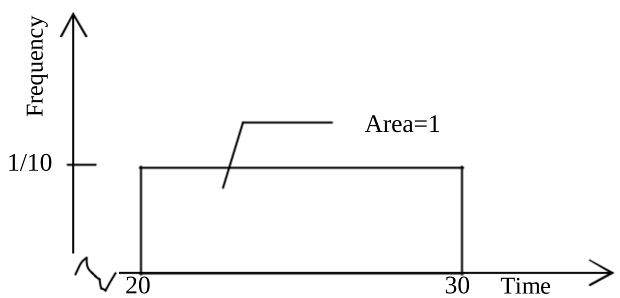
So the ratings are

- Poor-36420
- Good-43879
- Excelent-25821

QUESTION 3

a) Probability distributions are approximations of observed frequency distributions

i) Uniform distribution- here a random variable takes any value in a continuous interval, say a and b, in such a way that no value is more likely than the other.
Example- a contractor footing from Kibera takes 20 to 30 minutes to walk to work in industrial area.



b

a

$$\text{Mean} = \frac{a+b}{2} = \frac{20+30}{2} = 25$$

$$\text{Varaince} = \frac{(b-a)^2}{12}$$

$$\text{Probability } P(c<x<d) = \frac{d-c}{b-a}$$

For example, if c=23 and d=27 then $P(23<x<27)=\frac{27-23}{30-20} = \frac{4}{10} = \frac{2}{5}$

ii) Poison distribution describes probabilities for a number of occurrences at random point in time or space given by the following probability mass function

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$ Where λ-mean or expected number of successes per unit time

x-random variable (number of successes)

Number of occurrence in any interval is independent of the number of occurrence in other intervals.
Probability of occurrence is same throughout the field of observation.
Probability of more than one occurrence at any point is approximately zero.
Mean is equal to the variance, which equal the expected number of successes per unit time.

iii) Normal distribution has a bell shaped density curve which is based on the following probability density function.

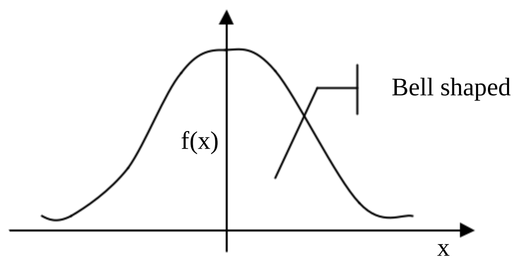
$$N(\mu,\sigma)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma}} \text{ and } \infty<x<\infty$$

Where σ-standard deviation

μ-mean

x-random variable

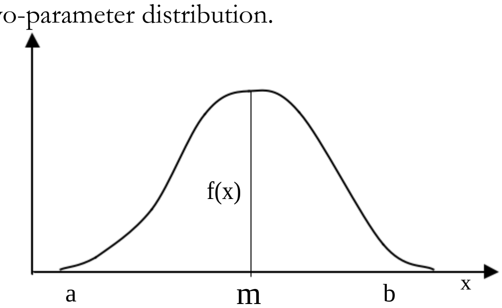
Standard normal distribution can be obtained from normal distribution N(μ,σ) to be $z=\frac{x-\mu}{\sigma}$ whose values are usually tabulated in tables.



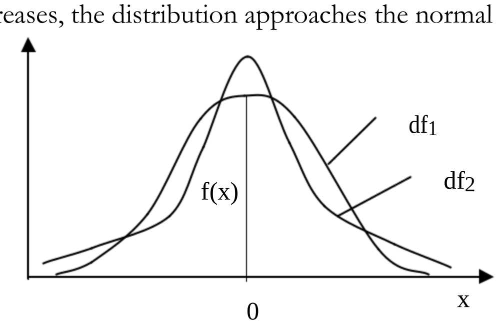
iv) Beta distribution is a probability distribution that has finite limits, is unimodal and can assume flexible shapes. It is two-parameter distribution.

$$\text{Mean}=\frac{a+4m+b}{6}$$

$$\text{Variance}=\left(\frac{b-a}{6}\right)^2$$



v) t-distribution is similar to normal distribution (bell shaped density function) but the shape depends on the parameter degree of freedom (df). DF is the number of independent random observations in a sample. As *df* increases, the distribution approaches the normal distribution



H_0- $\mu_{before} = \mu_{after}$ Training did not improve performance

H_1- $\mu_{before} < \mu_{after}$ Training improved performance

First it is necessary to know whether the population variances are equal or not. So the null and alternative hypothesis to test this is a follows:

H_0- $\sigma_1^2 = \sigma_2^2$ The population variances are equal

H_1- $\sigma_1^2 \neq \sigma_2^2$ The population variances are not equal

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{70}{12} = 5.833$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{77}{12} = 6.42$$

Employee	Before x_1	$(x_1 - \bar{x}_1)^2$	After x_2	$(x_2 - \bar{x}_2)^2$
A	4	3.4	5	2.0
B	5	1.4	4	11.7
C	3	24.1	6	0.5
D	7	5.4	8	10.0
E	8	23.5	7	1.7
F	6	0.2	5	12.0
G	5	4.9	9	46.7
H	9	80.2	9	53.4
I	10	156.3	10	115.6
J	6	0.3	6	1.7
K	4	37.0	4	64.2
L	3	96.3	4	70.1
	70	**432.8**	**77**	**389.7**

So $s_1^2 = \frac{\sum(x_1 - \bar{x}_1)^2}{n_1} = \frac{432.8}{12} = 36.06$

$s_2^2 = \frac{\sum(x_2 - \bar{x}_2)^2}{n_2} = \frac{389.7}{12} = 32.48$

Using 5% level of significance and 11 degree of freedom

$$F = \frac{n_1(n_2-1)s_1^2}{n_2(n_1-1)s_2^2} = \frac{12\times 11}{12\times 11} \times \frac{s_1^2}{s_2^2} = \frac{s_1^2}{s_2^2} = \frac{36.06^2}{32.48^2} = 1.23$$

From F-distribution tables $F_{5\%\ 11,11}=2.82$

Since calculated F is less than $F_{5\%\ 11,11}$, then the two population variances are equal.

So to test the first hypothesis,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{5.833 - 6.75}{\sqrt{\frac{(12 \times 36.06 + 12 \times 32.48)}{12+12-2}\left(\frac{1}{12} + \frac{1}{12}\right)}} = \frac{-0.917}{2.496} = -0.367$$

From t-distribution tables $t_{5\%,22}=-1.72$

Since calculated t =0.367 is more than $t_{5\%,22}=-1.72$then the null hypothesis is accepted. This means that the training did not improve performance.

QUESTION 4

a) Bias is a non-sampling error (error occurring during the process of gathering data) that is systematic and deprives a statistical result of its representativeness. It is cumulative as the sample size increases rather than balancing out like random errors.

Ways that bias is introduced include:

- Deliberate selection based on personal judgment of what is representative

- Having substitute members or units that are convenient rather than because of difficulty in making contact with given units.
- Failure to cover the whole of the selected sample like when some people do not answer questioners
- Haphazard selection by human beings rather than being random
- Inadequate, hasty, incomplete and misleading interviewing.

Sampling bias is tolerated in some investigations. Unbiased sample is one that is free from selection and procedural bias.

b) H_0:p=0.45 The proportion of dissatisfied is true for the whole population
H_1:p>0.45 Larger proportion of technicians is dissatisfied than is true for the whole population The test statistic is the standardized normal distribution

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.65 - 0.45}{\sqrt{\frac{0.45 \times 0.55}{100}}} = \frac{0.2}{0.0497} = 4.02$$

Where: $\hat{p}$ - Sample proportion of dissatisfied
P - Population proportion of dissatisfied n - number of sample
q - population proportion of satisfied = 1-P

Since calculated z=4.02> $z_{5\%}$=1.645 from normal distribution tables the null hypothesis is rejected and the conclusion is that a larger proportion of technicians is dissatisfied than is true for the whole population.

c) H_0- $\mu_1 = \mu_2$ There is a no significant difference in number of incorrect records H_1- $\mu_1 \neq \mu_2$ There is a significant difference in number of incorrect records μ_1-production mean of incorrect records
μ_2-sales mean of incorrect records
First it is necessary to know whether the population variances are equal H_0- $\sigma_1^2 = \sigma_2^2$ The population variances are equal
H_1- $\sigma_1^2 \neq \sigma_2^2$ The population variances are not equal

$$F = \frac{n_1(n_2-1)s_1^2}{n_2(n_1-1)s_2^2} = \frac{10\times 4}{5 \times 9} \times \frac{7^2}{6^2} = \frac{1960}{1620} = 1.2099$$

Given that n_1=10
n_2=5
s_1=7
s_2=6

Since calculated F=1.2099< $F_{5\%,9,4}$=5.999 from F-distribution tables, then the null hypothesis is accepted that the variance are equal. The test statistic is therefore as follows:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{45 - 55}{\sqrt{\left(\frac{10\times 45 + 5\times 25}{10+5-2}\right)\left(\frac{1}{10} + \frac{1}{5}\right)}} = \frac{-10}{3.774} = -2.65$$

Since calculated t=-2.65<$t_{5\%,13\,df}$=-2.16 then the null hypothesis is rejected. This means that there is a significant difference in the number of incorrect records in production and sales.

QUESTION 5

a) Estimated standard error of a regression equation is the error in approximating a given equation for given data relationship. It is expressed as follows for a bivariate distribution.

$$S_e = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}}$$

Where y-observed values of dependent variable
x-the observed values of the independent variable
n-the number of pairs of the variables
a,b-the the constant and slope of the regression equation

The denominator is to take care of the approximation using already approximated values a and b, thereby reducing the number of independent variables by 2.

b)

i) The table to assist in determination of regression equation is as follows

	Size x	Price y	xy	x^2	y^2
	2.30	4.20	9.66	5.29	17.64
	2.40	4.50	10.80	5.76	20.25
	3.00	4.80	14.40	9.00	23.04
	2.60	4.90	12.74	6.76	24.01
	3.60	5.30	19.08	12.96	28.09
	3.00	6.20	18.60	9.00	38.44
	2.70	7.50	20.25	7.29	56.25
	3.60	8.50	30.60	12.96	72.25
	2.00	3.50	7.00	4.00	12.25
Total	**25.20**	**49.40**	**143.13**	**73.02**	**292.22**

$\text{Price} = a + b \times \text{size}$ in million shillings (size in'000' square meters)

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \frac{9\times 143.13 - 25.2\times 49.4}{9\times 73.02 - (25.2)^2} = 1.955$$

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n} = \frac{49.4}{9} - 1.955\times\frac{25.2}{9} = 0.0141$$

So $\text{Price} = 0.0141 + 1.955\times \text{size}$

ii) The relationship between size and price is that there is a minimum price to be charged of Sh. 14,091 then for every increase of 1,000 square meter Sh. 1.955 million is added.

iii) The equation coefficient ‘a’ represents the minimum charge for land while ‘b’ represents the additional charge per every 1,000 square meters.

iv) The estimated standard error of regression equation is

$$S_e = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}} = \sqrt{\frac{292.22 - 0.0141\times 49 - 1.955\times 143.13}{9-2}} = 1.293$$

v) The price when the size of land is 2,150 square meters is

$\text{Price} = 0.0141 + 1.955\times 2.15 = 4.21735$ million shillings

QUESTION 6

a)

i) Sensitivity analysis on the objective function coefficients checks by how much a coefficient can vary before the existing optimal solution is no longer optimal.
Sensitivity analysis on the right hand side (RHS) constants checks by how much a constraint can vary before the optimal solution is no longer feasible.

ii) Linear programming is a mathematical model that seeks to optimise a multivariable function given a number of constraints and its assumptions of linearity, certainty, divisibilty and additivity
Transportation problem is a where the main aim is to allocate supply at each origin so as to optimise a criterion while satisfying the demand at each destination
Assignment problem is a special transportation problem that has the objective of optimally allocating n origins or activities to n destination or needs.

b)

i) Let s_1, s_2,s_3 be the slack in resources A,B,C so that the objective function and constraints are as follows in the standard form.
Maximise $Z + x_1 + x_2 - 3x_3 + 2x_4 + 0\,s_1 + 0\,s_2 + 0\,s_3 = 0$ Profit

Subject to:

$x_1 + 3x_2 - x_3 + 2x_4 + s_1 + 0\,s_2 + 0\,s_3 = 7$ Resource A

$-x_1 - 2x_2 - 4x_4 + 0s_1 + s_2 + 0\,s_3 = 12$ Resource B

$-x_1 + 4x_2 + 3x_3 + 8x_4 + 0s_1 + 0\,s_2 + s_3 = 10$ Resource C

Where $s_j, x_j \geq 0$, j=1,2,3,4

Table 1										
	X_1	X_2	X_3	X_4	S_1	S_2	S_3	solution	ratio	
S_1	1	3	-1	2	1	0	0	7	-7	
S_2	-1	-2	0	4	0	1	0	12	∞	
S_3	-1	4	3	8	0	0	1	10	10/3	⇐
z	1	1	-3	2	0	0	0	0	0	
Table 2			⇑							
S_1	2/3	13/3	0	14/3	1	0	1/3	31/3		
S_2	-1	-2	0	4	0	1	0	12		
X_3	-1/3	4/3	1	8/3	0	0	1/3	10/3		
z	0	5	0	10	0	0	1	10		

The iteration is stopped here since the z row contains non-negative values.

Notes:
Putting s_1, s_2 and s_3 as the basic solution makes the table 1. The coefficients of variables x_1 , x_2, x_3, x_4 in each of the three resource equations are put in the appropriate rows as indicated. The limits are the start of the solution. The coefficients of the function to be maximised are also put in the last row.
The most negative coefficient (for a maximisation problem) is chosen to identify the key column and shows the incoming variable that will improve the solution the most)
Then the solution column is divided by the coefficients in the key column to obtain the ratio column.
Then the lowest positive ratio (non-negative and non-infinity) is identified to give the key row. The element in the intersection of the key row and key column is the key element (in this case 3)
Every element in the key row is divided by the key element to come up with the new row 3. To obtain the other new rows, row operations are done with the objective to make all the coefficients in the key column be zero while the key element remains one.
For example to obtain the row 1 of table 2 (with aim of making element –1 be zero in the key column; row 3table 1+ row 1 table 1.
The other row operations are:
Row2 table 2= row2 table 1(because coefficients in the key column is already zero)
Row4 table 2= 3×row3 table 2 + row4 table 1.
This whole process (iteration) is then repeated for the next table until there is no negative coefficient in the last row. In this case we stop the iteration. The solution can then be read from the basis column and solution column. Sensitivity analysis can also be done.

ii) Optimal solution is $x_3=10/3$ $x_2=0$ $x_1=0$, $x_4 = 0$ and the optimum
z=10 $Z=c_1x_1+c_2x_2+c_3x_3+c_4x_4$

iii) Let Δ_2 be a change in x_2 coefficient, so that $c_2=(1+\Delta_2)$. Since x_2 is not part of the basis of the solution the following is true about the coefficient. The values are added to the coefficients to arrive at the final value in this case 5. So having Δ_2 added to the coefficient means that the final value will be $5+\Delta_2$. The values in the z row should be more or equal than zero to ensure the variable does not enter into the basis.

$5+\Delta_2 \geq 0 \Rightarrow \Delta_2 \geq -5$ Since $c_2=(1+\Delta_2)$, then the range for c_2 is as follows $c_2 \geq -4$

QUESTION 7

a)

		Hankol				
		1	2	3	4	Row minima
EAA	1	5000	-2000	12000	-5000	-5000
	2	6000	2000	7000	6000	2000 ⇐Maximum
	3	-2000	0	-4000	7000	-4000
Column maxima		6000	2000	12000	7000	
			⇑ Minimum			

There is a saddle point. The strategy to follow is 2 for both EAA and Hankol. The value of the game will be 2,000.

b)

i) The payoff table is as follows

		Demand						
	Probability	0.02	0.08	0.1	0.4	0.2	0.15	0.05
		15	**20**	**25**	**30**	**35**	**40**	**45**
Production	**15**	90	90	90	90	90	90	90
	20	-130	120	120	120	120	120	120
	25	-350	-100	150	150	150	150	150
	30	-570	-320	-70	180	180	180	180
	35	-790	-540	-290	-40	210	210	210
	40	-1010	-760	-510	-260	-10	240	240
	45	-1230	-980	-730	-480	-230	20	270

Notes:
The payoff table is constructed as follows:
Production quantities are put on the rows and demand quantities on the columns in multiples of 5 kg.
Now, if 15 kg are produced and 15kg are demanded:
Then the payoff=sales- cost of sales=15×50-15×44=Sh90
If 15 kg are produced and 20 kg are demanded:
Then the payoff=sales- cost of sales=15×50 -15×44=Sh90 This will be the same throughout the row. On the other hand if extra is produced than is demanded like if 20kg is produced and only 15 kg are demanded:
Then the payoff=sales- cost of sales=15×50-20×44=-Sh130 (which is a loss)
If 30kg are produced and only 20kg are demanded:
Then the payoff=sales- cost of sales=20×50-30× 44=-Sh320
The rest of the payoffs on the table are obtained the same way.

ii) Expected payoff for each production level = $\sum\left(\text{Payoff}_{i,j} \times P_i\right)^b$ where i-given demand and j-given production For example at production level 15:

Expected payoff=90×0.02+90×0.08+90×0.1+90×0.4+90×0.2+90×0.15+90×0.05=90

Production	Expected Payoff
15	90
20	115
25	120 ⇐Maximum
30	100
35	-20
40	-190
45	-397.5

So the production level that maximises expected payoff is 25 kg per day.

QUESTION 8

a)

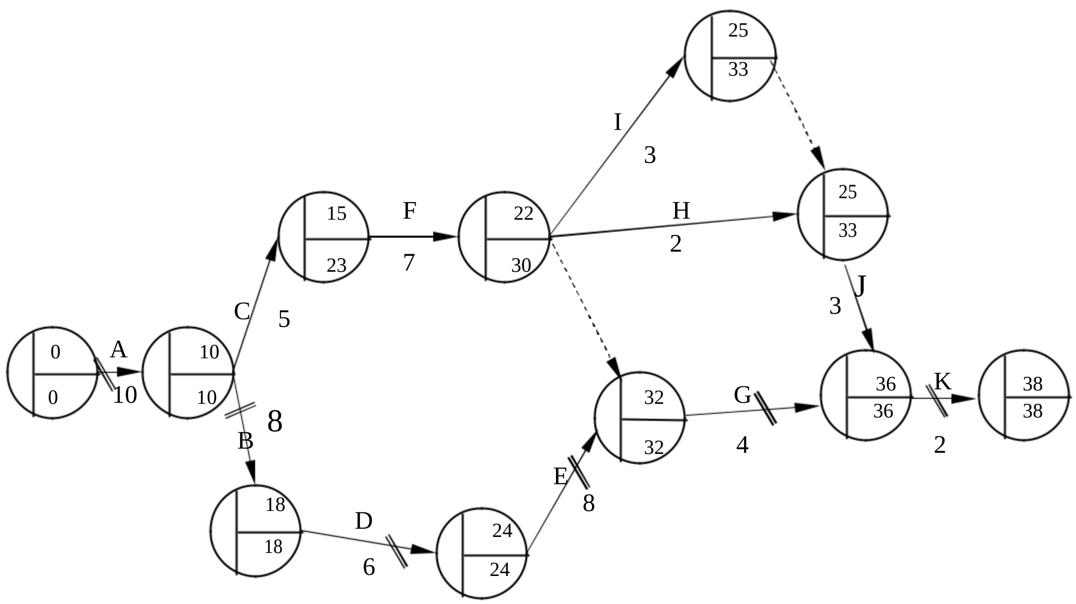
i) Latest start time is the last time that an activity can start and be completed on time without delaying subsequent activities.

ii) Earliest start time is the first time an activity can start immediately after the preceding activity

iii) Is the shifting of non-critical activities between earliest and latest starting times to reduce the maximum number of resource required assuming that units of resources are perfect substitutes.

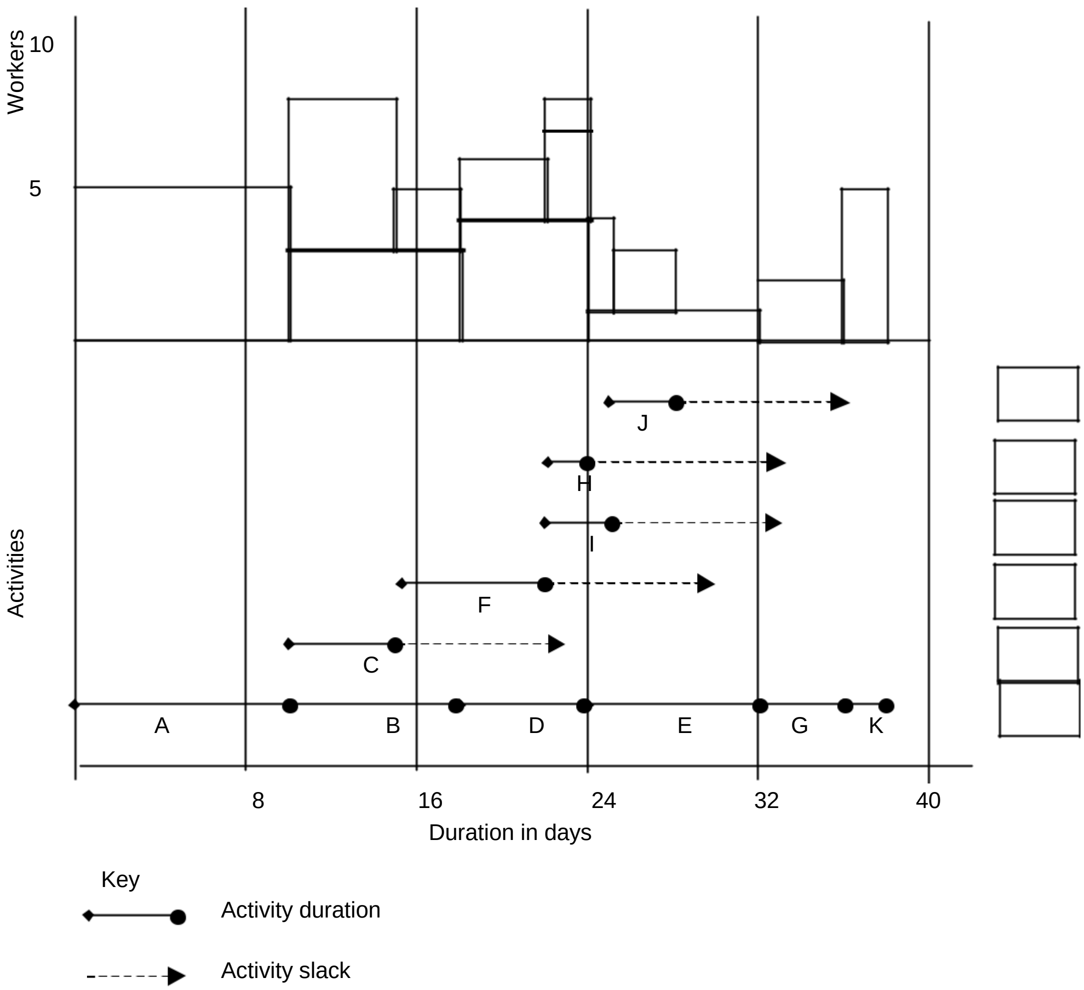
b)

i)



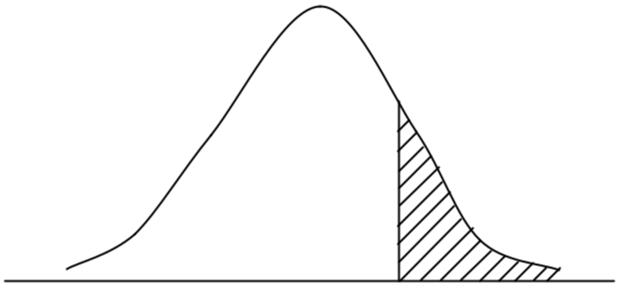
Critical path A-B-D-E-G-K

ii)



iii) The work team resource cannot be levelled further because of inflexibilities in the latest and earliest times of non-critical activities work team requirement.

Statistical Tables

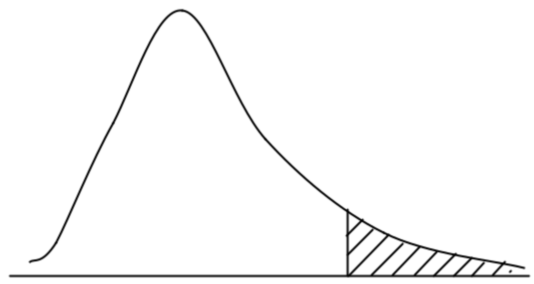


t table with right tail probabilities

df\p	0.4	0.25	0.1	0.05	0.025	0.01	0.005	0.0005
1	0.32492	1	3.077684	6.313752	12.7062	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.5407	5.84091	12.924
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.94318	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.306	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.36343	1.795885	2.20099	2.71808	3.10581	4.437
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.681	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.34503	1.76131	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.75305	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.015
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.25658	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.31946	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.68485	1.317836	1.710882	2.0639	2.49216	2.79694	3.7454
25	0.25606	0.68443	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75	3.646
inf	0.253347	0.67449	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Area between 0 and z

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0	0.004	0.008	0.012	0.016	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.483	0.4834	0.4838	0.4842	0.4846	0.485	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.489
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.492	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.494	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.496	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.497	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.498	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.499	0.499

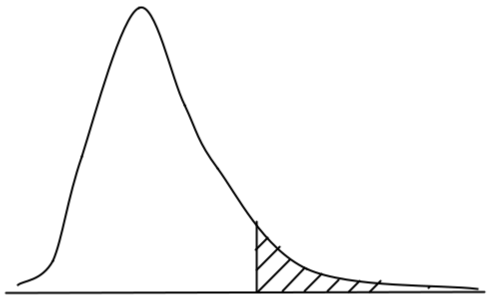


χ^2

Right tail areas for the *Chi-square* Distribution

df\area	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636		11.0705012	.8325015.08	62716.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790 2	.70039 3	.32511 4.	16816 5.8	9883 8.3	4283 11.3	887514.6	836616.9	189819.0	227721.6	659923.5	8935
10	2.15586 2	.55821 3.2	4697 3.9	4030 4.86	518 6.73	720 9.341	82 12.54	88615.98	71818.30	70420.48	31823.20	92525.188	18
11	2.60322	3.05348 3	.81575 4	.57481 5.5	7778 7.5	8414 10.3	410013.7	006917.2	750119.6	751421.9	200524.7	249726.7	5685
12	3.07382	3.57057 4	.40379 5	.22603 6.3	0380 8.4	3842 11.3	403214.8	454018.5	493521.0	260723.3	366626.2	169728.2	9952
13	3.56503	4.10692 5	.00875 5	.89186 7.0	4150 9.2	9907 12.3	397615.9	839119.8	119322.3	620324.7	356027.6	882529.8	1947
14	4.07467	4.66043 5	.62873 6	.57063 7.7	8953 10	.1653113	.3392717	.1169321	.0641423	.6847926	.1189529	.1412431.3	1935
15	4.60092	5.22935 6	.26214 7	.26094 8.5	4676 11	.0365414	.3388618	.2450922	.3071324	.9957927	.4883930	.5779132.8	0132
16	5.14221	5.81221 6	.90766 7	.96165 9.3	1224 11	.9122215	.3385019	.3688623	.5418326	.2962328	.8453531	.9999334.2	6719
17	5.69722	6.40776 7	.56419 8	.67176 10	.0851912	.7919316	.3381820	.4886824	.7690427	.5871130	.1910133	.4086635	.71847
18	6.26480	7.01491 8	.23075 9	.39046 10	.8649413	.6752917	.3379021	.6048925	.9894228	.8693031	.5263834	.8053137	.15645
19	6.84397	7.63273 8	.90652 10	.1170111	.6509114	.5620018	.3376522	.7178127	.2035730	.1435332	.8523336	.1908738	.58226
20	7.43384 8	.26040 9.5	9078 10.	8508112.	4426115.	4517719.	3374323.	8276928.	4119831.	4104334.	1696137.	5662339.9	9685
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
26	11.16024	12.19815	13.84390	15.37916	17.29188	20.84343	25.33646	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74940	26.33634	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196

F Table for alpha=.10 .



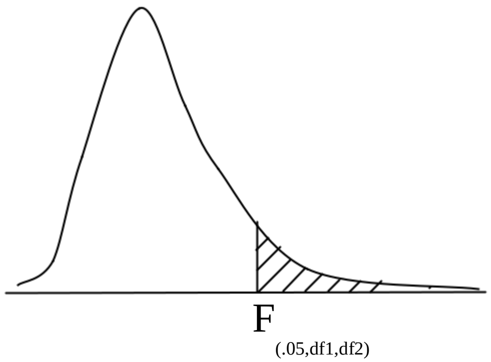
F

(.10,df1,df2)

df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	INF
1					39.8634649.50000 53.5932455.8329657.24008 58.2044258.90595 59.4389859.85759 60.1949860.70521 61.2203461.74029 62.00205 62.2649762.52905 62.7942863.06064 63.32812														
2	8.52632	9.00000	9.16179	9.24342	9.29263	9.32553	9.34908	9.36677	9.38054	9.39157	9.40813	9.42471	9.44131	9.44962	9.45793	9.46624	9.47456	9.48289	9.49122
3	5.53832	5.46238	5.39077	5.34264	5.30916	5.28473	5.26619	5.25167	5.24000	5.23041	5.21562	5.20031	5.18448	5.17636	5.16811	5.15972	5.15119	5.14251	5.13370
4	4.54477	4.32456	4.19086	4.10725	4.05058	4.00975	3.97897	3.95494	3.93567	3.91988	3.89553	3.87036	3.84434	3.83099	3.81742	3.80361	3.78957	3.77527	3.76073
5	4.06042	3.77972	3.61948	3.52020	3.45298	3.40451	3.36790	3.33928	3.31628	3.29740	3.26824	3.23801	3.20665	3.19052	3.17408	3.15732	3.14023	3.12279	3.10500
6	3.77595	3.46330	3.28876	3.18076	3.10751	3.05455	3.01446	2.98304	2.95774	2.93693	2.90472	2.87122	2.83634	2.81834	2.79996	2.78117	2.76195	2.74229	2.72216
7	3.58943	3.25744	3.07407	2.96053	2.88334	2.82739	2.78493	2.75158	2.72468	2.70251	2.66811	2.63223	2.59473	2.57533	2.55546	2.53510	2.51422	2.49279	2.47079
8	3.45792	3.11312	2.92380	2.80643	2.72645	2.66833	2.62413	2.58935	2.56124	2.53804	2.50196	2.46422	2.42464	2.40410	2.38302	2.36136	2.33910	2.31618	2.29257
9	3.36030	3.00645	2.81286	2.69268	2.61061	2.55086	2.50531	2.46941	2.44034	2.41632	2.37888	2.33962	2.29832	2.27683	2.25472	2.23196	2.20849	2.18427	2.15923
10	3.28502	2.92447	2.72767	2.60534	2.52164	2.46058	2.41397	2.37715	2.34731	2.32260	2.28405	2.24351	2.20074	2.17843	2.15543	2.13169	2.10716	2.08176	2.05542
11	3.22520	2.85951	2.66023	2.53619	2.45118	2.38907	2.34157	2.30400	2.27350	2.24823	2.20873	2.16709	2.12305	2.10001	2.07621	2.05161	2.02612	1.99965	1.97211
12	3.17655	2.80680	2.60552	2.48010	2.39402	2.33102	2.28278	2.24457	2.21352	2.18776	2.14744	2.10485	2.05968	2.03599	2.01149	1.98610	1.95973	1.93228	1.90361
13	3.13621	2.76317	2.56027	2.43371	2.34672	2.28298	2.23410	2.19535	2.16382	2.13763	2.09659	2.05316	2.00698	1.98272	1.95757	1.93147	1.90429	1.87591	1.84620
14	3.10221	2.72647	2.52222	2.39469	2.30694	2.24256	2.19313	2.15390	2.12195	2.09540	2.05371	2.00953	1.96245	1.93766	1.91193	1.88516	1.85723	1.82800	1.79728
15	3.07319	2.69517	2.48979	2.36143	2.27302	2.20808	2.15818	2.11853	2.08621	2.05932	2.01707	1.97222	1.92431	1.89904	1.87277	1.84539	1.81676	1.78672	1.75505
16	3.04811	2.66817	2.46181	2.33274	2.24376	2.17833	2.12800	2.08798	2.05533	2.02815	1.98539	1.93992	1.89127	1.86556	1.83879	1.81084	1.78156	1.75075	1.71817
17	3.02623	2.64464	2.43743	2.30775	2.21825	2.15239	2.10169	2.06134	2.02839	2.00094	1.95772	1.91169	1.86236	1.83624	1.80901	1.78053	1.75063	1.71909	1.68564
18	3.00698	2.62395	2.41601	2.28577	2.19583	2.12958	2.07854	2.03789	2.00467	1.97698	1.93334	1.88681	1.83685	1.81035	1.78269	1.75371	1.72322	1.69099	1.65671
19	2.98990	2.60561	2.39702	2.26630	2.17596	2.10936	2.05802	2.01710	1.98364	1.95573	1.91170	1.86471	1.81416	1.78731	1.75924	1.72979	1.69876	1.66587	1.63077
20	2.97465	2.58925	2.38009	2.24893	2.15823	2.09132	2.03970	1.99853	1.96485	1.93674	1.89236	1.84494	1.79384	1.76667	1.73822	1.70833	1.67678	1.64326	1.60738

21	2.96096	2.57457	2.36489	2.23334	2.14231	2.07512	2.02325	1.98186	1.94797	1.91967	1.87497	1.82715	1.77555	1.74807	1.71927	1.68896	1.65691	1.62278	1.58615
22	2.94858	2.56131	2.35117	2.21927	2.12794	2.06050	2.00840	1.96680	1.93273	1.90425	1.85925	1.81106	1.75899	1.73122	1.70208	1.67138	1.63885	1.60415	1.56678
23	2.93736	2.54929	2.33873	2.20651	2.11491	2.04723	1.99492	1.95312	1.91888	1.89025	1.84497	1.79643	1.74392	1.71588	1.68643	1.65535	1.62237	1.58711	1.54903
24	2.92712	2.53833	2.32739	2.19488	2.10303	2.03513	1.98263	1.94066	1.90625	1.87748	1.83194	1.78308	1.73015	1.70185	1.67210	1.64067	1.60726	1.57146	1.53270
25	2.91774	2.52831	2.31702	2.18424	2.09216	2.02406	1.97138	1.92925	1.89469	1.86578	1.82000	1.77083	1.71752	1.68898	1.65895	1.62718	1.59335	1.55703	1.51760
26	2.90913	2.51910	2.30749	2.17447	2.08218	2.01389	1.96104	1.91876	1.88407	1.85503	1.80902	1.75957	1.70589	1.67712	1.64682	1.61472	1.58050	1.54368	1.50360
27	2.90119	2.51061	2.29871	2.16546	2.07298	2.00452	1.95151	1.90909	1.87427	1.84511	1.79889	1.74917	1.69514	1.66616	1.63560	1.60320	1.56859	1.53129	1.49057
28	2.89385	2.50276	2.29060	2.15714	2.06447	1.99585	1.94270	1.90014	1.86520	1.83593	1.78951	1.73954	1.68519	1.65600	1.62519	1.59250	1.55753	1.51976	1.47841
29	2.88703	2.49548	2.28307	2.14941	2.05658	1.98781	1.93452	1.89184	1.85679	1.82741	1.78081	1.73060	1.67593	1.64655	1.61551	1.58253	1.54721	1.50899	1.46704
30	2.88069	2.48872	2.27607	2.14223	2.04925	1.98033	1.92692	1.88412	1.84896	1.81949	1.77270	1.72227	1.66731	1.63774	1.60648	1.57323	1.53757	1.49891	1.45636
40	2.83535	2.44037	2.22609	2.09095	1.99682	1.92688	1.87252	1.82886	1.79290	1.76269	1.71456	1.66241	1.60515	1.57411	1.54108	1.50562	1.46716	1.42476	1.37691
60	2.79107	2.39325	2.17741	2.04099	1.94571	1.87472	1.81939	1.77483	1.73802	1.70701	1.65743	1.60337	1.54349	1.51072	1.47554	1.43734	1.39520	1.34757	1.29146
120	2.74781	2.34734	2.12999	1.99230	1.89587	1.82381	1.76748	1.72196	1.68425	1.65238	1.60120	1.54500	1.48207	1.44723	1.40938	1.36760	1.32034	1.26457	1.19256
inf	2.70554	2.30259	2.08380	1.94486	1.84727	1.77411	1.71672	1.67020	1.63152	1.59872	1.54578	1.48714	1.42060	1.38318	1.34187	1.29513	1.23995	1.16860	1.00000

F Table for alpha=.05 .



df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	INF
1	161.4476	199.5000	215.7073	224.5832	230.1619	233.9860	236.7684	238.8827	240.5433	241.8817	243.9060	245.9499	248.0131	249.0518	250.0951	251.1432	252.1957	253.2529	254.3144
2	18.5128	19.0000	19.1643	19.2468	19.2964	19.3295	19.3532	19.3710	19.3848	19.3959	19.4125	19.4291	19.4458	19.4541	19.4624	19.4707	19.4791	19.4874	19.4957
3	10.1280	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5264
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.6581	5.6281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.3650
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.8780

20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660	2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.8380	1.7831
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.8128	1.7570
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7896	1.7330
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401	1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000